

Preliminary Design of Learning the Lowest Common Multiple and Greatest Common Factor Using the RADEC Learning Model

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Abstract. Learning obstacles were found in students who studied LCM and GCF. This research aims to develop an initial design in the form of a hypothetical learning trajectory for LCM and GCF using the RADEC (Read, Answer, Discuss, Explanation, and Create) learning model which is based on obstacles learning findings. This research is part of the design research method. The design research method consists of preliminary design, teaching experiment, and retrospective analysis. The focus of this research is the preliminary design stage. Based on the results, it is known that students have difficulties in solving questions on LCM and GCF material, namely that students cannot use the concepts of LCM and GCF to solve problems, students tend to think that the concepts of LCM and GCF are only factor trees or procedural factorization, and students do not understand the concepts of LCM and GCF. To overcome these learning obstacles, a hypothetical learning trajectory for the LCM and GCF was prepared using the RADEC learning model which was divided into three meetings. The first meeting was about multiples, common multiple, and LCM. The second meeting was about factors, common factors, and GCF. The third meeting was regarding determining the LCM and GCF using prime factorization. Each learning flow from each meeting is described in the form of HLT which consists of learning objectives, learning activities, and student thinking estimates. The theoretical implications in this research are the new findings of HLT LCM and GCF with the RADEC learning model and practical implications in this research are that HLT can be used as a consideration for teachers in designing LCM and GCF learning by paying attention to students' learning trajectories.

Keywords: Design Research; GCF; LCM; RADEC.

1. Introduction

Students study mathematics from elementary school through university level courses. Learning mathematics in elementary school becomes the basis for students studying mathematics at the next level. Mathematics learning in elementary school plays an important role in forming rational, critical, careful, as well as effective, and efficient thinking. This is because elementary school students know mathematics as a tool that they can use to solve numeracy problems in everyday life. These basic mathematical abilities include problem-solving abilities, reasoning abilities, communication abilities, connection abilities, and representation abilities (Karsenty, 2020).

Mathematics learning aims to form students who can understand mathematical concepts, link concepts, use reasoning and generalize, solve mathematical problems, communicate mathematical ideas, and have an attitude of appreciating the usefulness of mathematics (Nuraini et al., 2022). Apart from that, mathematics learning in elementary school includes three aspects, namely numbers, geometry and measurement, and data processing or statistics. These aspects are related to each other to form students' comprehensive understanding of mathematics. Mathematics lessons themselves consist of symbols, calculations, and abstract concepts. Therefore, many students have difficulty learning mathematics (Yantoro et al., 2021). One of the difficulties students have in studying mathematics is the material on the Lowest Common Multiple (LCM) and Greatest Common Factor (GCF) in elementary school.

1.1. Problem Statement

One of the topics covered in elementary school mathematics is LCM and GCF. LCM is the smallest equal value resulting from two or more multiples of a number, and GCF is the largest value resulting from two or more number factors. Problem-solving on the concepts of the LCM and GCF in mathematics learning is difficult for students to pass well (Bintaş & Çamlı, 2009). This is because the concepts of the LCM and GCF are still very procedurally taught in elementary schools using factor trees and tables without reviewing the concepts of these methods (Fauzan et al., 2020). Therefore, Students do not yet comprehend the steps involved in solving problems in the LCM and GCF materials, nor are they familiar with problem-solving questions (Meilani & Maspupah, 2019). Apart from that, students have difficulty learning LCM and GCF material due to students' lack of understanding of teacher explanations, resulting in low student mathematics learning outcomes (Latifah et al., 2020).

One of the factors causing students' difficulties in studying LCM and GCF material is teaching materials (Marsena et al., 2021). The teaching materials used must facilitate students' learning trajectories so that they can anticipate the emergence of problems in learning. A learning trajectory has three parts, namely 1) specific mathematics learning goals, 2) the path that students go through to achieve these goals, and 3) learning activities that support achieving these goals (Clements & Sarama, 2020). Learning trajectories support teachers in modeling student thinking, identifying student learning needs, and interacting with students during learning (Rohimah, et al., 2022). Therefore, it is important to design and develop learning trajectories for learning LCM and GCF material that will be implemented. This is done to anticipate the emergence of learning obstacles that arise in the process of learning LCM and GCF material, as well as to improve learning outcomes.

1.2. Related Research

The RADEC learning model can have a significant impact on teachers' abilities in making learning plans (Handayani et al., 2019). Learning planning activities are important because good planning will produce a good learning process (Nurgiyantoro, 2010). Learning design aims to smooth the flow of student learning and provide solutions to anticipate the emergence of learning obstacles experienced by students. Therefore, LCM and GCF learning with the RADEC model is an alternative learning design to overcome student learning obstacles which can facilitate student learning trajectories.

Related research regarding the RADEC model or regarding LCM and GCF material is as follows: 1) the application of the RADEC learning model in mathematics learning in elementary schools can overcome learning difficulties faced by students thereby increasing elementary school students' mathematical understanding abilities (Nugraha & Prabawanto, 2021), 2) The use of the RADEC learning model in LCM and GCF material can help and make it easier for teachers to provide correct explanations of learning material (Yuza et al., 2023), and 3) the RADEC learning model also influences mathematical critical thinking skills (Pratama et al., 2020; Yuliany et al., 2023) and on the mathematics learning outcomes of elementary school students (Ramadhani et al., 2023).

In general, previous studies mostly created media, teaching aids, or multimedia to overcome difficulties in studying LCM and GCF (Patan & Martinez, 2018; A'yun & Rahmawati, 2018; Khairiyah, 2019; Yensy, 2020; Alim et al., 2020; Fauzan et al., 2020). In this research, a Hypothetical Learning Trajectory (HLT) will also be created to study LCM and GCF material so that there are no difficulties for students in learning it. This research tries to design how to learn LCM and GCF material using the RADEC learning model. The specific question studied is what the student's learning trajectory is in studying LCM and GCF material in elementary schools using the RADEC learning model. The novelty of this research is the learning obstacles found in the LCM and GCF material, as well as the development of a hypothetical learning trajectory in LCM and GCF learning with the RADEC learning model.

1.3. Research Objectives

Developing and designing teaching materials with an HLT design is needed to minimize the difficulties experienced by students in learning using the RADEC learning model. Based on this,

this research aims to design the initial design of LCM and GCF materials using the RADEC learning model. The initial design in this research was the development of a hypothetical learning trajectory in LCM and GCF learning with the RADEC learning model which was prepared based on obstacle learning findings.

2. Theoretical Framework

2.1. RADEC Learning Model

As times progress, learning in schools must be adapted to 21st-century learning, higher-order thinking Skills (HOTS) learning, character learning, and multiliteracy learning. The way to develop learning that trains multiliteracy and builds character is by using a learning model that is appropriate to the learning objectives and Indonesian context. One model that suits this is the Read, Answer, Discuss, Explain, and Create (RADEC) learning model. The RADEC Learning Model was chosen because it accommodates students' needs in honing character readiness, multiliteracy abilities, and 21st-century skills. The RADEC learning model is a learning model initiated by Sopandi (2017) which is an abbreviation for effective learning, Read, Answer, Discuss, Explain, and Create.

The Read, Answer, Discuss, Explain, Create (RADEC) learning methodology puts students at the center of the learning process and aligns with 21st-century abilities (Sopandi, 2019). The RADEC learning model was developed by Sopandi (2017) with four main foundations, namely First, founded on the national education system's objectives, which include helping pupils reach their full potential as human beings with faith, moral character, good health, knowledge, ability, creativity, independence, and responsibility. Second, learning resources, whether in the form of books or on the internet, are easy for students to obtain, so that students can obtain information independently before learning in class. Third, to optimize the development of students' cognitive abilities in interaction with the social environment. Fourth, to develop students' literacy skills and train students to answer questions and discuss with friends. Apart from that, students can also practice writing skills when proposing ideas or writing reports on solving problems or projects they have worked on.

The stages of the RADEC learning model are the first stages Read, students read all the information needed about the material before class starts (Sopandi, 2017). Students independently search for information from various sources, both from textbooks and the internet. This is to develop the character of a lover of reading, responsibility, perseverance, and tenacity, as well as develop students' literacy skills. In the second stage, Answer, students are given pre-learning questions to measure thinking abilities from reasoning to problem-solving. In this way, students can find out the extent of their abilities in understanding reading at the previous stage. In the third stage, Discuss, students discuss the answers to pre-learning questions with a group of friends. At this stage, the teacher ensures that students actively communicate in groups and obtain correct answers. In the fourth stage, explain, students explain the important points of the material in front of the class. The teacher encourages students to ask questions, provide input, and add to or refute the presentation. The fifth stage, Create, is where students think of a creative idea that might be created from the material they have studied. The five stages of RADEC support and direct students in developing an understanding of the concepts of the LCM and GCF, not only procedurally but also conceptually.

The characteristics of the RADEC learning model are 1) instruction that consistently motivates pupils to participate fully in the learning process; 2) encourage students to learn independently; 3) help students to connect what is known with what is being studied; 4) connecting lesson material with real life; 5) provide opportunities for students to actively ask questions, discuss and conclude learning; and 6) Give students the chance to delve deeply into the content by assigning pre-learning tasks.

2.2. Lowest Common Multiple (LCM) and Greatest Common Factor (GCF)

The Lowest Common Multiple (LCM) is a multiple of numbers that is the result of multiplying that number by a positive integer, while the Greatest Common Factor (GCF) is a dividing factor of

numbers that completely divides that number (Aqeel, 2024). Before learning about LCM, students are first introduced to the concept of multiples and multiples of commonality. Two or more numbers can have an infinite number of common multiples, so in the LCM students only look for the smallest common multiple. Likewise, with GCF material, students are first introduced to factors and common factors. Two or more numbers can have many common factors, so in GCF students only choose the one number that is the largest of the common factors. Apart from that, understanding the concept of prime numbers is also used in finding the LCM and GCF with prime number factorization to break down the given number into numbers with their prime factors and then use equations to solve (Patan & Martinez, 2018).

This research is related to the LCM and GCF with the first material focusing on multiples, partnership multiples, and LCM. The second material concerns factors, common factors, and GCF. The third meeting was about determining the LCM and GCF using prime numbers. Solving the LCM and GCF can be done using a factor tree or table.

3. Method

3.1. Research Design

This research is part of design research. Design research is a cyclic process of creating designs or testing learning activities and other aspects of design (Gravemeijer, 2012). The steps in design research are: (1) preliminary design, (2) teaching experiment, and (3) retrospective analysis. The focus of this research is only on the preliminary design stage. The preliminary design is designing learning on LCM and GCF material using the RADEC learning model by reviewing several kinds of literature, analyzing learning videos, analyzing teacher and student books, as well as compiling the characteristics of difficulties and learning obstacles for researchers to use in designing a series of learning activities containing Hypothetical Learning Trajectory (HLT).

3.2. Participant/Respondent

Analysis of student learning barriers was carried out in three state elementary schools in the city of Bandung, class V, totaling 74 students with an age range of 10-11 years, by giving tests and interviews. The results of the study and analysis of obstacles are used to design student learning activities that include predictions of learning trajectories.

3.3. Data Collection

The first stage of this research was to conduct a literature review related to LCM and LCM material by analyzing textbooks, learning videos, and research results related to LCM and LCM learning. Next, the researcher identified the obstacles experienced by students in studying the LCM and GCF material. Learning obstacles data was obtained from test results and interviews with respondents. The test given is in the form of five essay questions with the details of the questions as follows in Figure 1.

1. Amel installed lights in his room with two different colored lights, namely red and blue. The red light comes on every 3 minutes and the blue light comes on every 4 minutes. Amel turned on the red light at 20.00 WIB and the blue light at 20.15 WIB. At what time will both lights come on together?
2. Fina has 18 cakes which will be put into the cake box. What are the possible numbers in the box and the number of cakes in it?
3. Mrs. Siti, Mrs. Dini, and Mrs. Neni like to shop at Gedebage market. Mrs. Siti goes shopping every 4 days, Mrs. Dini goes shopping every 6 days, and Mrs. Neni goes shopping every 8 days. If on October 5 2022 they go shopping together, on what date will they go together again?
4. Ali has 24 pens and 16 books. The pens and books are wrapped and then distributed to orphans in orphanages.
 - a. How many packages can Ali make?
 - b. How many pens and books are in each package?

Figure 1. Problems regarding the LCM and GCF that have been given to students.

3.4. Data Analysis

Data analysis was carried out by triangulating the data. Data triangulation is a data validity checking technique that uses something other than the data for checking purposes or as a comparison to the data (Moleong, 2022). The triangulation used in this research is comparing test result data with interview data and comparing interview results with the results of analysis of learning videos and textbooks. Therefore, the results of data analysis produced an LCM and GCF learning design using the RADEC learning model and written in the form of a Hypothetical Learning Trajectory (HLT). This Hypothetical Learning Trajectory (HLT) is dynamic and developing and can be revised in teaching experimental activities.

3.5. Validity and Reliability

Describe the quality of the instrument or the validity of the data according to the research method and design. Internal validity in design research is based on the quality of data collection and data interpretation that leads to conclusions. Data collection in this research involves more than one type of data, which allows data triangulation to occur, thereby increasing internal validity in data analysis. The instruments used in this research were tested for content and face validity by several experts in mathematics education. Apart from that, this instrument was tested first in small groups before the implementation stage. Furthermore, external validity, according to (Moleong, 2021), external validity focuses on the results obtained in different situations guided by questions about how certain elements of the results obtained will apply to other situations.

Reliability in design research is carried out qualitatively in two ways (Widjaja, 2008), namely data triangulation and cross-interpretation. (1) Data triangulation, involving different data sources to see the relationship between various sources, including learning videos, test results, and interviews. (2) Cross-interpretation, namely asking for consideration from experts (supervisors) to provide advice regarding the data obtained to minimize researcher subjectivity in interpreting research data.

4. Findings

The results of this research are divided into two parts, namely the obstacles experienced by students in studying LCM and GCF material and hypothetical learning trajectories to overcome these obstacles.

4.1. Obstacles of LCM and GCF

According to the findings of the analysis conducted on each problem, students encountered difficulties when attempting to solve the LCM and GCF problems because they were unable to apply the concepts to solve problems; instead, they tended to think of the concepts only in terms of factor trees or procedural factorization, as demonstrated by the students' response to question number 4 below.

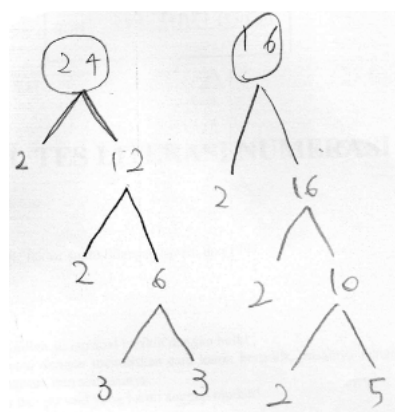


Figure 2. Students' answers in answering question number 4

In Figure 2 above, students answer question number 4 using a factor tree. However, students made a mistake in writing their factor tree and there was no conclusion from the results of the factor tree. This understanding makes students limited to the context and does not yet understand conceptually. In order to overcome and foresee the appearance of learning hurdles, students must have learning experiences that help them comprehend the concepts found in the LCM and GCF.

4.1. Hypothetical Learning Trajectory

As indicated in Table 1, the findings of this exploratory study are utilized to create learning that includes hypothetical learning trajectories for LCM and GCF content. The three parts of the HLT are the learning objectives, the learning exercises that students complete, and the student thought conjecture.

Table 1. Hypothetical Learning Trajectory for Learning LCM and GCF Use RADEC Learning Model

Learning Objectives	Learning Activities	Student Thinking Conjecture
<ul style="list-style-type: none"> • Students can explain the concepts of multiples, common multiples, and Lowest Common Multiple (LCM). • Students can solve problems related to multiples, common multiples, and LCM using the parsing method. 	<p>Pre-learning Activities</p> <ol style="list-style-type: none"> 1. Read stage Students read books, teaching materials, and other reading materials related to multiples, common multiples, and LCM. 2. Answer stage Students complete the prepared worksheet by responding to pre-learning questions about multiples, common multiples, and LCM. <p>Learning Activities</p> <ol style="list-style-type: none"> 3. Discuss stage <ul style="list-style-type: none"> • Students are divided into 5 heterogeneous groups. • Students confer to decide how to respond to pre-learning questions and finish worksheet activities. 4. Explain stage <ul style="list-style-type: none"> • After students share the findings of their discussion, other groups are asked to reply, offering both arguments in favor of and against them. • Students answer probing questions from the teacher (according to student needs). • Students listen to the teacher's explanation (according to student needs). 5. Create stage <ul style="list-style-type: none"> • Students agree on creative ideas related to solving problems with the concept of multiples, multiples, and LCM using the parsing method. 	<ul style="list-style-type: none"> • Reading sources that students read can be textbooks, teaching materials, and sources from the internet. • Through comprehension of reading materials, students seek solutions to pre-learning queries. • Students use the internet to search for answers to pre-learning questions. • Students discuss with their groups the various student answers at the previous pre-learning stage. • Students debate and settle on responses to pre-learning queries. • Students present the results of their group discussions. • Students provide feedback on the group presentation. • Students answer probing questions. • Students listen to the teacher's explanation. • Students apply the concepts of multiples, multiples, and LCM in practice questions. • Students come up with original solutions for multiples, LCM, and multiples problems. • Students reach consensus on original solutions to multiples, LCM, and multiples issues by employing the parsing method.

Learning Objectives	Learning Activities	Student Thinking Conjecture
<ul style="list-style-type: none"> Students can explain the concept of factors, common factors, and the Greatest Common Factor (GCF). Students can solve problems related to factors, common factors, and GCF using the parsing method. 	<p>Pre-learning Activities</p> <ol style="list-style-type: none"> Read stage Students read books, teaching materials, and other reading materials related to factors, common factors, and GCF. Answer stage Students answer pre-learning questions related to factors, common factors, and GCF on the worksheets that have been prepared. <p>Learning Activities</p> <ol style="list-style-type: none"> Discuss stage <ul style="list-style-type: none"> Students are divided into 5 heterogeneous groups. Students confer to decide how to respond to pre-learning questions and finish worksheet activities. Explain stage <ul style="list-style-type: none"> After students share the findings of their discussion, other groups are asked to reply, offering both arguments in favor of and against them. Students answer probing questions from the teacher (according to student needs) Students listen to the teacher's explanation (according to student needs). Create stage <ul style="list-style-type: none"> Students agree on creative ideas related to solving problems with the concept of solving factors, common factors, and GCF problems using the parsing method. 	<ul style="list-style-type: none"> Reading sources that students read can be textbooks, teaching materials, and sources from the internet. Through comprehension of reading materials, students seek solutions to pre-learning queries. Students use the internet to search for answers to pre-learning questions. Students discuss with their groups the various student answers at the previous pre-learning stage. Students debate and settle on responses to pre-learning queries. Students present the results of their group discussions Students provide feedback on the group presentation. Students answer probing questions. Students listen to the teacher's explanation. Students apply the concepts of factors, common factors, and GCF in practice questions. Students find creative ideas related to solving factors, common factors, and GCF problems. Students agree on creative ideas related to solving factors, common factors, and GCF problems using the parsing method.
<ul style="list-style-type: none"> Students can determine the LCM and GCF using prime factorization (prime numbers, prime factors, prime factorization, factor tree method, and 	<p>Pre-learning Activities</p> <ol style="list-style-type: none"> Read stage <ul style="list-style-type: none"> Students read books, teaching materials, and other reading materials related to LCM and GCF material using prime factorization. Answer stage <ul style="list-style-type: none"> Students respond to pre-learning questions inquiries about the application of the concepts of LCM and GCF with prime factorization on the worksheet that has been prepared. 	<ul style="list-style-type: none"> Reading sources that students read can be textbooks, teaching materials, and sources from the internet. Through comprehension of reading materials, students seek solutions to pre-learning queries. Students use the internet to search for answers to pre-learning questions. Students discuss with their groups the various student answers at the previous pre-learning stage.

Learning Objectives	Learning Activities	Student Thinking Conjecture
<p>table/swallow method).</p> <ul style="list-style-type: none"> Students can solve problems related to the concepts of LCM and GCF using prime factorization. 	<p>Learning Activities</p> <ol style="list-style-type: none"> Discuss Stage <ul style="list-style-type: none"> Students are divided into 5 heterogeneous groups. Students confer to decide how to respond to pre-learning questions and finish worksheet activities. Explain stage <ul style="list-style-type: none"> After students share the findings of their discussion, other groups are asked to reply, offering both arguments in favor of and against them. Students answer probing questions from the teacher (according to student needs). Students listen to the teacher's explanation (according to student needs). Create Stage <ul style="list-style-type: none"> Students agree on creative ideas related to LCM and GCF with prime factorization to solve problems related to everyday events. 	<ul style="list-style-type: none"> Students debate and settle on responses to pre-learning queries. Students present the results of group discussions. Students respond to the group presentation. Students answer probing questions. Students listen to the teacher's explanation Students apply the concepts of LCM and GCF with prime factorization in practice questions. Students find creative ideas related to LCM and GCF with prime factorization. Students agree on creative ideas related to LCM and GCF with prime factorization.

5. Discussion

The learning challenges that students have when studying the LCM and GCF content in this study are consistent with a number of earlier studies that found students struggled to comprehend questions and find solutions when it came to the LCM and GCF (Meilani & Maspupah, 2019), difficulties in determining solutions to story problems (Mufidah et al., 2021), difficulties in using multiplication, factors, and natural number calculation operations in solving problems related to LCM and GCF (Mahmud et al., 2023).

This learning challenge arises from the fact that the instructional resources students utilize to learn are still primarily procedural, requiring a thinking shift in how they approach studying LCM and GCF (Desriyati et al., 2015). Students' thinking is not bridged from basic comprehension starting from the ideas of factors, multiples, LCM, and GCF since they learn LCM and GCF procedurally from abstract things (A'yun & Rahmawati, 2018). Students are given example problems using factor trees and imitate how to work on example problems to solve the next problem (Khairiyah, 2019). The factor tree technique is the only way the teacher offers to solve problems; additional approaches are not provided (Hadi, 2015). Based on this, the learning provided by this teacher has an impact on students' non-conceptual understanding.

These obstacles cause low student mathematics learning outcomes, especially in LCM and GCF material. Efforts to overcome these learning obstacles are by holding remedial, enrichment, carrying out frequent exercises, and using varied learning media (Rohimah et al., 2022). Apart from that, these obstacles need to be anticipated by designing teaching materials or learning designs that can facilitate students' learning trajectories. Efforts to facilitate students' learning flow provide a solution to anticipate the emergence of learning obstacles that students experience. The learning process that considers the learning trajectory in this research is built by the Hypothetical Learning Trajectory (HLT).

HLT refers to teachers' predictions regarding student learning trajectories in a learning process. HLT is hypothetical because the actual learning trajectory cannot be known before learning is carried out. Therefore, teachers are regularly involved in modifying HLT. HLT is also used to improve students' understanding of mathematics and as a vehicle for planning mathematics learning (Simon & Tzur, 2012). The learning trajectory provided by the teacher in HLT is a selection of special learning designs so that it is the teacher's best design in the learning process. HLT consists of three components, namely learning objectives that determine the direction of learning, learning activities, and students' conjecture thinking (Clements & Sarama, 2020). The RADEC learning model, which has stages for reading, responding, discussing, explaining, and producing, is used in the creation of HLT. The five RADEC stages assist in developing strategies to overcome and foresee the formation of learning obstacles in the design of HLT on LCM and GCF materials.

Learning LCM and LCM in class V elementary school is a continuation of the material on multiples and factors that students have studied in class IV elementary school. Therefore, learning begins with students determining multiples, common multiples, and LCM. Instilling the concept of multiples begins with contextual problems regarding multiplying two natural numbers and calculating the product. Next, students are directed to recognize the concept of common multiples with contextual problems related to multiples of two natural numbers and finding the same multiples. The concept of LCM is studied by students based on contextual problems related to multiples of two numbers, finding the common multiple of two numbers, and determining the smallest common multiple. Therefore, this learning flow plays a role in instilling the LCM concept in students' understanding abilities.

At the second meeting, the material discussed was factors, common factors, and GCF. The implantation of concepts starting from factors is presented from contextual problems related to multiplications of natural numbers that have the same result, groups of numbers that have the same product, and numbers that divide evenly. After instilling the concept of factors, students learn the concept of common factors, namely from contextual problems related to the factors of two natural numbers and the same factors. The same factors that have been studied previously provide students with an opportunity to understand the concept of GCF, namely from contextual problems related to the factors of two natural numbers, the common factor of two natural numbers, and the largest common factor. Naming the GCF concept becomes students' capital in solving problems related to GCF.

The final meeting, namely the third meeting, was regarding determining the LCM and GCF using prime factorization. Learning begins with the concept of prime numbers which is studied from contextual problems regarding number factors, Erathosthenes' sieve technique, as well as the factors of the number 1 and the number itself. After that, students learn the concept of prime factors from contextual problems related to number factors and prime number factors. Next, students are introduced to the concept of prime factorization from contextual problems regarding prime factors of several numbers, multiplications of prime factors, and numbers in the form of multiplication of prime numbers. Students can find out how to determine the LCM and GCF using prime factorization by using a factor tree or a table. Therefore, students understand strong concepts first before using factor trees or tables.

In general, the learning process using the RADEC learning model consists of five stages, namely the read stage, answer stage, discuss stage, explain stage, and create stage. Before learning begins, the first stage of the RADEC learning model is reading, namely, students are assigned to read books, teaching materials, and other reading materials related to the material that will be discussed in the lesson. This aims to guide students in understanding the learning information that will be carried out and guided by learning questions. These questions cover various levels of questions, from lower-order thinking skills to higher-order thinking skills. This reading activity is carried out as a tool for students to improve their literacy skills (Pohan et al., 2020).

After carrying out reading activities at the read stage, students answer pre-learning questions at the second stage of the RADEC learning model, namely answer. Students answer pre-

learning questions based on the knowledge they have acquired independently. Students answering pre-learning questions aim to practice students' knowledge regarding the essential parts of the material they must master. This is to familiarize students with exploring information from various sources independently and to help teachers identify the needs of different students (Sopandi, 2019). Teachers are not only tasked with delivering material but are also able to develop students' potential, one of which is by creating learning questions (Kelana et al., 2022).

The third stage in the RADEC learning model is discussed. Students are formed into several groups to discuss their answers to the pre-learning questions. The teacher encourages students who have successfully understood the pre-learning questions to discuss and provide guidance to other students who do not understand. This activity requires students to discuss answers between groups and other groups. This stage supports students in developing higher-order thinking skills (HOTS), namely, students can analyze the learning material studied (Pratama et al., 2020).

The fourth stage in the RADEC learning model is explained. Students make presentations after students discuss at the discussion stage. Presentations are made by representatives of each group who are deemed capable of explaining the material well enough to be presented in front of the class. The study materials presented include cognitive learning indicators that have been designed for the implementation of learning (Kusumaningpuri & Fauziati, 2021). The teacher facilitates students in providing questions, rebuttals, and suggestions to group representatives who are presenting or completing material that has not been exposed (Yuza et al., 2023). At this stage, the teacher also provides explanations with the help of video media and PowerPoint displays to overcome students' lack of understanding.

The final stage in the RADEC learning model is creation. Create is the final step in a series of read-to-explain activities which aims for students to be able to create or formulate creative ideas. Teachers can inspire students to create creative ideas that emerge in LCM and GCF learning. Creative ideas can be formulated as productive questions, problem identification, or conclusions. This activity encourages students to think creatively, work together, and communicate in groups to find creative ideas, identify ideas that will be realized, plan the realization process, and carry out the realization. Based on the five stages of the RADEC learning model above, the learning objectives of LCM and GCF material can be achieved by implementing the RADEC learning model (Yuza et al., 2023).

The results of this research can be new findings in HLT LCM and GCF using the RADEC learning model. Apart from that, HLT LCM and GCF using the RADEC learning model is the selection of a learning design that can overcome or anticipate the emergence of learning difficulties and obstacles. The implications of the results of this research are practically used as considerations for mathematics teachers to design learning on LCM and GCF material by paying attention to students' learning trajectories. The weakness of this research is that the development of HLT was limited to a sample of elementary school students, there may be several characteristics that could be added if the sample was completed from middle and high school students as well as tertiary institutions. Suggestions for further research are to continue research at the teaching experiment stage with the note that teachers must master the material and be able to overcome various problems that occur when learning takes place so that students do not experience obstacles; Teachers must be patient in guiding and directing students so that they can generate ideas from within themselves. Teachers must prepare sufficient time for students to be able to carry out each stage of the RADEC learning model.

6. Conclusion

This research concludes that students experience several obstacles in learning LCM and GCF material which can be overcome or anticipated with learning design using the RADEC learning model. Based on the results of preliminary research, it is known that students have difficulties in solving questions on LCM material, namely that students cannot use the concepts of LCM and

LCM to solve problems, students tend to think that the concepts of LCM and LCM are only factor trees or procedural factorization, and students do not understand the concepts of LCM and GCF. To overcome these learning obstacles, a hypothetical learning trajectory for the Corruption Eradication Committee and GCF was prepared using the RADEC learning model which was divided into three meetings. The first meeting was about multiples, multiples of alliances, and the LCM. The second meeting was about factors, common factors, and GCF. The third meeting was regarding determining the LCM and GCF using prime factorization. Each learning flow from each meeting is described in the form of an HLT consisting of learning objectives, learning activities, and student thinking estimates. The initial designs of LCM and GCF developed in this research are expected to be able to overcome or anticipate the emergence of learning difficulties and obstacles so that mathematics learning objectives can be achieved well. Furthermore, the results of this research can be developed and implemented at the teaching experiment stage, where retrospective analysis and revising of the design are conducted.

Limitation

The limitation of this research is that the analysis of learning barriers obtained in this research is limited to one grade level in elementary school. This analysis of learning obstacles regarding the LCM and GCF can be developed from the answers from middle school, high school, or college-level students to get views related to each other. Therefore, HLT can be designed with more guesswork on students' thinking.

Recommendation

Future research can analyze learning barriers using samples of middle school and college students. The results of this research can be used for implementation at the teaching experiment stage, and HLT can be redeveloped from the teaching experiment results.

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Conflict of Interest

Concerning the publishing of this work, the author states that there is no conflict of interest.

References

- Alim, J. A., Sari, I. K., Alpusari, M., Sulastio, A., Mulyani, E. A., Putra, R. A., & Hermita, N. (2020). Interactive Multimedia Development on LCM and GCF Material. *Journal of Physics: Conference Series*, 1655(1), 012090. <https://doi.org/10.1088/1742-6596/1655/1/012090>
- Aqeel, I. A. (2024). *Improving Methods of Finding the Least Common Multiple and the Greatest Common Divisor Using Conceptual Map*. 21(S3), 1468–1479. <https://migrationletters.com/index.php/ml/article/view/7482>
- A'yun, N. Q., & Rahmawati, I. (2018). Pengembangan Media Interaktif Si Pontar Berbasis Aplikasi Android Materi KPK dan FPB Mata Pelajaran Matematika Kelas IV SD [Development of Si Pontar Interactive Media Based on Android Applications for LCM and GCF Mathematics Subjects for Grade IV Elementary School]. *Jurnal Penelitian Pendidikan Guru Sekolah Dasar*, 6(2), 254773. <https://ejournal.unesa.ac.id/index.php/jurnal-penelitian-pgsd/article/view/23410>
- Bintaş, J., & Çamlı, H. (2009). The effect of computer-aided instruction on students' success in solving LCM and GCF problems. *Procedia-Social and Behavioral Sciences*, 1(1), 277–280. DOI: <https://doi.org/10.1016/j.sbspro.2009.01.050>
- Clements, D. H., & Sarama, J. (2020). *Learning and teaching early math: The learning trajectories approach*. Routledge.

- Desriyati, W., Mashadi, M., & Gemawati, S. (2015). Cara Lain Menentukan FPB dan KPK [Other Ways to Determine LCM and GCF]. *Jurnal Sains Matematika Dan Statistika*, 1(1), 52–55. DOI: <http://dx.doi.org/10.24014/jsms.v1i1.1973>
- Fauzan, A., Yerizon, Y., & Yulianti, D. (2020). The RME-based local instructional theory for teaching LCM and GCF in primary school. *Journal of Physics: Conference Series*, 1554(1), 012078. DOI: <http://dx.doi.org/10.1088/1742-6596/1554/1/012078>
- Fitri Nuraini, T., Cahyani Haditia, N., Rahman Hakim, A., & Indah Sari, N. (2022). Fostering Students' Mathematical Communication Skills in Mathematics Learning. In *ISEJ: Indonesian Science Education Journal* (Vol. 3, Issue 1). <https://siducat.org/index.php/isej/article/view/593>
- Gravemeijer, K. (2012). Local instruction theories as means of support for teachers in reform mathematics education. In *Hypothetical Learning Trajectories* (pp. 105–128). Routledge. DOI: https://doi.org/10.1207/s15327833mtl0602_3
- Hadi, S. (2015). Scaffolding dalam Menyelesaikan Permasalahan KPK dan FPB [Scaffolding in Resolving LCM and GCF Problems]. *Jurnal Pendidikan Matematika*, 1(1). DOI: <https://doi.org/10.21154/ibriez.v1i1.16>
- Handayani, H., Sopandi, W., Syaodih, E., Setiawan, D., & Suhendra, I. (2019). Dampak perlakuan model pembelajaran radec bagi calon guru terhadap kemampuan merencanakan pembelajaran di sekolah dasar [The impact of the radec learning model treatment for prospective teachers on the ability to plan learning in elementary schools]. *Pendas: Jurnal Ilmiah Pendidikan Dasar*, 4(1), 79–93. DOI: <https://doi.org/10.23969/jp.v4i1.1857>
- Karsenty, R. (2020). Mathematical ability. *Encyclopedia of Mathematics Education*, 494–497.
- Kelana, J. B., Sopandi, W., Firdaus, A. R., Maulana, Y., Fasha, L. H., & Fiteriani, I. (2022). Kemampuan Guru Sekolah Dasar dalam Membuat Pertanyaan Pra Pembelajaran Menggunakan Model RADEC [Elementary School Teachers' Ability to Create Pre-Learning Questions Using the RADEC Model]. *Jurnal Cakrawala Pendas*, 8(4), 1171–1180. DOI: <https://doi.org/10.31949/jcp.v8i4.2688>
- Khairiyah, U. (2019). Respon siswa terhadap media dakon matika materi KPK dan FPB pada siswa kelas IV di SD/MI Lamongan [Student responses to dakon matika media, LCM and GCF material in class IV students at SD/MI Lamongan]. *Jurnal Studi Kependidikan Dan Keislaman*, 5(2), 197–204. DOI: <http://dx.doi.org/10.53627/jam.v5i2.3476>
- Kusumaningpuri, A. R., & Fauziati, E. (2021). Model pembelajaran RADEC dalam perspektif filsafat konstruktivisme Vygotsky [The RADEC learning model in the perspective of Vygotsky's constructivism philosophy]. *Jurnal Papeda: Jurnal Publikasi Pendidikan Dasar*, 3(2), 103–111. DOI: <https://doi.org/10.36232/jurnalpendidikandasar.v3i2.1169>
- Latifah, N., Wakhyudin, H., & Cahyadi, F. (2020). Miskonsepsi Penyelesaian Soal Cerita Matematika Materi KPK dan FPB Sekolah Dasar [Misconceptions about Solving Mathematics Story Problems for Elementary School LCM and GCF Material]. *Jurnal Riset Pendidikan Dasar*, 3(2), 181–195. <https://doi.org/10.26618/jrpd.v3i2.4078>
- Mahmud, M. R., Turmudi, T., Sopandi, W., Rohimah, S. M., & Pratiwi, I. M. (2023). Learning obstacles analysis of lowest common multiple and greatest common factor in primary school. *Jurnal Elemen*, 9(2), 440–449. DOI: <https://doi.org/10.29408/jel.v9i2.12359>
- Marsena, N., Fauzan, A., & Gistituati, N. (2021). Development of Problem-Based Mathematics Teaching Materials for Elementary School Students. *International Journal of Education Dynamics*, 3(2), 41–47. <https://doi.org/10.24036/ijeds.v3i2.303>
- Meilani, M., & Maspupah, A. (2019). Analisis kemampuan pemecahan masalah SD pada materi KPK dan FPB [Analysis of elementary school problem solving abilities in LCM and GCF material]. *Journal On Education*, 2(1), 25–35. DOI: <https://doi.org/10.31004/joe.v2i1.264>

- Moleong, L. J. (2021). *Metodologi Penelitian Kualitatif [Qualitative Research Methodology]*. Bandung: PT Remaja Rosdakarya.
- Mufidah, M., Akina, A., & Fauziah, S. (2021). Kesalahan Siswa dalam Menyelesaikan Soal Cerita FPB dan KPK di Sekolah Dasar [Students' Mistakes in Solving LCM and GCF Story Questions in Elementary Schools]. *Jurnal Kreatif Online*, 9(2), 111–118. <https://garuda.kemdikbud.go.id/documents/detail/3184869>
- Nugraha, T., & Prabawanto, S. (2021). The Enhancement of Students' Mathematical Conceptual Understanding Through RADEC Learning Model. *Eduma: Mathematics Education Learning and Teaching*, 10(2), 167–177. DOI: <http://dx.doi.org/10.24235/eduma.v10i2.9073>
- Nurgiyantoro, B. (2010). *Penilaian pembelajaran sastra berbasis kompetensi [Competency-based literature learning assessment]*. Yogyakarta: BPF.
- Patan, R. & Martinez, R. (2018). Finding Greatest Common Factor and Least Common Multiple Using Alternative Method. *EPRA International Journal of Research and Development*, 3(6), 64-69. <https://eprajournals.com/IJSR/article/803>
- Pohan, A. A., Abidin, Y., & Sastromiharjo, A. (2020). Model Pembelajaran Radec Dalam Pembelajaran Membaca Pemahaman Siswa [Radec Learning Model in Learning Students' Reading Comprehension]. *Seminar Internasional Riksa Bahasa*, 250–258. <http://proceedings.upi.edu/index.php/riksabahasa/article/view/1354>
- Pratama, Y. A., Sopandi, W., Hidayah, Y., & Trihatusti, M. (2020). Pengaruh model pembelajaran RADEC terhadap keterampilan berpikir tingkat tinggi siswa sekolah dasar [The influence of the RADEC learning model on elementary school students' higher order thinking skills]. *JINoP (Jurnal Inovasi Pembelajaran)*, 6(2), 191–203. DOI: <https://doi.org/10.22219/jinop.v6i2.12653>
- Ramadhani, K., Witri, G., & Fendrik, M. (2023). Pengaruh Model Pembelajaran RADEC (Read, Answer, Discussion, Explaining and Create) Terhadap Hasil Belajar Matematika Siswa Kelas V SDN 194 Pekanbaru [The Influence of the RADEC Learning Model (Read, Answer, Discussion, Explaining and Create) on the Mathematics Learning Outcomes of Class V Students at SDN 194 Pekanbaru]. *El-Ibtidaiy: Journal of Primary Education*, 6(2), 190–199. DOI: <http://dx.doi.org/10.24014/ejpe.v6i2.23709>
- Rohimah, S. M., Darti, D., & Anggraeni, R. I. (2022). Analisis Learning Obstacles Pada Materi Pecahan Siswa Kelas IV Sekolah Dasar [Analysis of Learning Obstacles in Fraction Material for Class IV Elementary School Students]. *Symmetry: Pasundan Journal of Research in Mathematics Learning and Education*, 7(2), 171–180. DOI: <https://doi.org/10.23969/symmetry.v7i2.6386>
- Simon, M. A. & Tzur, R. (2012). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. In *Hypothetical learning trajectories* (pp. 91–104). Routledge.
- Sopandi, W. (2017). The quality improvement of learning processes and achievements through the read-answer-discuss-explain-and-create learning model implementation. *Proceeding 8th Pedagogy International Seminar*, 8, 132–139.
- Sopandi, W. (2019). Sosialisasi dan Workshop Implementasi Model Pembelajaran RADEC Bagi Guru-Guru Pendidikan dasar dan Menengah [Socialization and Workshop on Implementation of the RADEC Learning Model for Primary and Secondary Education Teachers]. *Pedagogia: Jurnal Pendidikan*, 8(1), 19–34. DOI: <https://doi.org/10.21070/pedagogia.v8i1.1853>
- Widjaja, W. (2008). *Local instruction theory on decimals: The case of Indonesian pre-service teachers*. University of Melbourne, Science and Mathematics Education Cluster.

- Yantoro, Y., Kurniawan, D. A., Perdana, R., & Rivani, P. A. (2021). A Survey of Process Skills Mathematics Learning in Elementary School. *Jurnal Pendidikan Dan Pengajaran*, 54(3). <https://doi.org/10.23887/jpp.v54i3.37180>
- Yensy, N. A. (2020). Metode Alternatif Menentukan KPK dan FPB Suatu Bilangan Bulat dengan Menggunakan Alat Peraga [Alternative Method for Determining the LCM and GCF of an Integer Using Visual Tools]. *PENDIPA Journal of Science Education*, 4(2), 107–114. <https://doi.org/10.33369/pendipa.4.2.107-114>
- Yuliany, N., Latuconsina, N. K., Abrar, A. I. P., & Wahyuni, I. (2023). Pengaruh Penerapan Model Pembelajaran RADEC (Read, Answer, Discuss, Explain, Create) terhadap Kemampuan Berpikir Kritis Matematis Peserta Didik [The Effect of Applying the RADEC Learning Model (Read, Answer, Discuss, Explain, Create) on Students' Mathematical Critical Thinking Ability]. *Al Asma: Journal of Islamic Education*, 5(2), 133–142. DOI: <https://doi.org/10.24252/asma.v5i2.41523>
- Yuza, A., Madona, A. S., Azkiya, H., Yulisna, R., & Harun, G. J. (2023). Validasi E-Modul Berbasis Radece Materi KPK Dan FPB Kelas IV Sekolah Dasar [Validation of Radece-Based E-Modules for LCM and GCF Material for Class IV Elementary Schools]. *JIPM (Jurnal Ilmiah Pendidikan Matematika)*, 11(2), 373–383. <https://garuda.kemdikbud.go.id/documents/detail/3581175>