p-ISSN 2355-5343
e-ISSN 2502-4795
http://ejournal.upi.edu/index.php/mimbar

Article Received: 28/01/2022; Accepted: 04/04/2022
Mimbar Sekolah Dasar, Vol. 9(1), 140-160
DOI: 10.53400/mimbar-sd.v9i1.43742

How Do Middle School Mathematics Teachers Conceptualize Open-Ended Questions?

Erhan Bingolbali✉[1] **& Hilmi Furkan Cevik**[2]

[1] Department of Mathematics and Science Education, School of Education, Afyon Kocatepe University, Afyonkarahisar, Turkey

[2] Ministry of National Education, Sanliurfa, Turkey

✉ ebingolbali@aku.edu.tr

Abstract. The study aims at examining middle school mathematics teachers' conceptions of open-ended questions. A questionnaire consisting of open-ended items was applied to 40 mathematics teachers. The teachers were asked to define the open-ended question in general and the mathematical open-ended question in particular and provide examples to exemplify their definitions. This study employs phenomenographic design aiming at revealing middle school mathematics teachers' conceptions and experience regarding open-ended questions in an exploratory manner. The findings show that the teachers explained the open-ended question through its form (appearance), the number of outputs, the process/method required, and its functionality. In addition, teachers defined the open-ended question mostly using non-mathematical terms, and they had particular difficulties defining the mathematical open-ended question. The teachers regarded questions with variable correct answers as open-ended, could not give examples of open-ended questions with infinitely correct answers, and some deemed closed-ended questions as open-ended. Although the participants were mathematics teachers, the examples they presented for the open-ended question were mostly from outside the field of mathematics. This study points out the fact that teachers need a guiding conceptualization of open-ended questions.

Keywords: open-ended questions; mathematical open-ended questions; teachers' conceptions; teachers' difficulties; mathematics teachers

How to Cite: Bingolbali, E., & Cevik, H. (2022). How Do Middle School Mathematics Teachers Conceptualize Open-Ended Questions?. *Mimbar Sekolah Dasar,* 9(1), 140-160. https://doi.org/10.53400/mimbar-sd.v9i1.43742.

INTRODUCTION ~ There is extensive literature on the use of open-ended questions, problems, and tasks for mathematics learning and teaching (Becker & Shimada, 1997; Nieminen et al., 2022; Nohda, 2000; OECD, 2017; Pehkonen, 1997). Compared to closed-ended questions, it is frequently stated that open-ended questions have the capacity to offer more ideas about the students' understanding, thinking, and knowledge (Silver, 1992). For this reason, it is often recommended to use open-ended questions as part of both teaching and assessment processes (Becker & Shimada, 1997; Nohda, 2000; OECD, 2017; Pehkonen, 1997). The emergence of the open approach (Nohda, 2000) or open-ended approach teaching methods (Becker & Shimada, 1997) can be considered a result of the importance given to open-ended questions. The same is the case for international exams ones, such as PISA and TIMSS also include open-ended questions as a part of students' assessment.

Examining open-ended questions in terms of student performances (Cai, 2000), the numbers of their inclusion in textbooks (Bingolbali, 2020a; Zhu & Fan, 2006), their use in the context of measurement-evaluation (MoNE, 2017; OECD, 2017), their contribution to the development of higher-order thinking skills (Kwon et al., 2006), their use in professional development programs (Zaslavsky, 1995) and their use as a teaching approach (Becker & Shimada, 1997; Nohda, 2000) are some examples of the study foci on the open-ended questions in the related literature (for details, see Bingölbali & Bingölbali, 2021). Although open-ended questions are examined in different research areas and their benefits for learning and teaching are frequently emphasized, the ways teachers conceptualize them are not sufficiently addressed in the related literature. Hence, this study specifically aims to examine Turkish middle school mathematics teachers' conceptions of open-ended questions and to determine how they conceptualize this question type in relation to closed-ended questions.

THEORETICAL FRAMEWORK

In describing open-ended tasks, the terms open-ended question, open-ended problem, and open-ended activity are used in the literature (Bingölbali & Bingölbali, 2021). While the similarities and differences between the concepts of a question, problem, and task are often not clearly expressed, placing the "open-ended" term before makes it more difficult to understand the meanings attributed to these concepts. In this section, a brief review of the related studies will be given to reveal both what the current state of definitions on open-ended questions is like and why an operational conceptualization is needed for them.

The related literature displays that different definitions and conceptualizations have been made for open-ended questions. According to Reitman (1966), in a problem consisting of 'the given, the operations, and the outcome', if 'the given' and 'the outcome' are provided, the problem is well-structured (closed-ended), but if at least one of them is not provided, this problem is not well-structured (open-ended) (Leung, 1997). Silver (1995) defined questions that allow for distinguished interpretations or different answers and questions that allow for a solution with different methods, as open-ended questions. Silver (1995) also described the problems that have not yet been solved in mathematics and the productive problems as open-ended. In addition, Aziza (2021) created a synthesis from the definitions of different researchers and defined the open-ended question as "a question that has more than one acceptable answer, can be solved through multiple solution methods, and can be developed into new questions". Furthermore, Pehkonen (1997) defined the open-ended problem as "investigations, problem posing, real-life situations, projects, problem fields (or problem sequences), problems without question, and problem variations ("what-if"-method)". His description suggests that he defines open-ended problems through problem situations that require active participation from students.

Bingölbali & Bingölbali (2020) defined open-ended and closed-ended questions based on the number of correct answers and the variability of these answers. Questions that have only one answer (e.g., what is the smallest prime number?) or a certain number of correct answers (e.g., writing all pairs of positive integers that add up to 20) are considered closed-ended. Questions that allow for finite variable correct answers (e.g., an example of an integer pair with a product of 50) or an infinite number of correct answers (e.g., giving an example of a rectangle with a perimeter of 20 cm) are described as open-ended. According to Bingölbali & Bingölbali (2020), if different solution methods are requested as output in a question, the question can be evaluated as closed or open-ended as above based on the number of outputs.

Open-ended questions have been the foci of research studies from different perspectives. These studies were classified under six themes by Bingölbali & Bingölbali (2021). These themes are;

a) Students' performance in open-ended questions and their views on these question types (Bingölbali & Bingölbali, 2021; Cai, 1995; Clarke et al., 1992; Sullivan et al., 1998, 2000; Sullivan & Clarke, 1992).
b) The use of open-ended questions as a teaching approach for teaching (Becker & Shimada, 1997; Lin et al., 2013; Nohda, 2000).
c) The frequencies and forms of open-ended questions in mathematics textbooks (Bingolbali, 2020b; Bingölbali & Bingölbali, 2020; Gracin, 2018; Han et al., 2011; Yang et al., 2017; Zhu & Fan, 2006).
d) The relationship between open-ended questions and higher-order thinking skills (e.g., creativity, divergent-convergent thinking) (Bennevall, 2016; Bingölbali & Bingölbali, 2020; Klavir & Hershkovitz, 2008; Kwon et al., 2006).
e) The use of open-ended questions in measurement and evaluation (İnceçam et al., 2018; McMillan, 2017; MoNE, 2017; OECD, 2017; Silver, 1992).
f) The use of open-ended questions for the professional development of teachers and teacher candidates (Bragg & Nicol, 2008; Zaslavsky, 1995).

As they are directly related to the purpose of this study, it is necessary to focus on the themes of measurement-evaluation and teacher-related studies. In terms of their use for measurement and evaluation, open-ended questions are characterized by the terms "constructed response items" and "extended response items" (Hogan & Murphy, 2007; İnceçam et al., 2018; McMillan, 2017). As they allow students to create their answers, open-ended questions are frequently associated with written examinations among practitioners and are also closely associated with tasks such as essays allowing students to make statements about the posed issues (McMillan, 2017). We have also witnessed in some official documents that even if they have only one correct answer, short-answer questions are considered open-ended questions as they require a written answer and do not have selected responses (ÖSYM, 2017).

As far as the conceptions of teachers on open-ended questions are concerned, it appears that the teachers themselves have not been the focus of much direct attention. In a study conducted with teachers, Pehkonen (1999) found that about half of the teachers did not provide an appropriate definition for an open-ended task and that only a quarter of the participants knew what an open-ended task was. Even though direct studies on teachers' conceptions of open-ended tasks are rare, indirect studies on teachers' ways of dealing with open-ended tasks are more encountered. Indirect studies are mostly about whether teachers use open-ended questions in classroom practices, how they do so, and their experiences of developing the questions for measurement-evaluation purposes (Aziza, 2021; Becker & Shimada, 1997; İnceçam et al., 2018; Kasar, 2013). In such a study, Kasar (2013) analyzed 67 lessons records of eight teachers (four middle school mathematics and four classroom teachers) in terms of alternative solutions and the types of questions they used. The findings revealed that 372 questions were addressed in the 67 lessons, and only six (1.6%) of them had multiple correct answers (open-ended), and 366 (98.4%) had one correct answer (closed-ended). Lastly, İnceçam et al. (2018) examined the open-ended item preparation competencies of middle school teachers, including mathematics teachers, revealed that teachers had difficulties in preparing open-ended question items.

As a result, when all these studies are considered all together, it is clear that there have been not sufficient studies on how teachers, who have a key role in the execution of teaching and learning activities, define and conceptualize open-ended questions. Based on this gap in the literature, this study aims to find answers to the following research questions:

- What are the conceptions of middle school mathematics teachers about open-ended questions?
- What kind of examples do middle school mathematics teachers provide for open-ended questions?

METHOD

Design

This qualitative research (Merriam & Tisdell, 2016) was designed as a phenomenographic study (Boulton-Lewis et al., 2001; Çekmez et al., 2012), which aimed at revealing middle school mathematics teachers' conceptions of and experience regarding open-ended questions in an exploratory manner. In this section, details about the participant group, the data collection tool, data collection process, and data analysis method are presented in order to reveal how this phenomenographic study was conducted.

Participants

This study was carried out by involving 40 (26 female and 14 male) middle school mathematics teachers from different provinces in Turkey. Participants were determined by the convenience sampling method (Yıldırım & Şimşek, 2011). Thirty-two of the participants were working mathematics teachers in state schools, and eight teachers were graduates waiting to be appointed. While 10% (4) of the participants had professional experience over five years, 70% (28) have 0-5 years of professional experience, and 20% (8) were graduates waiting to be appointed.

Data Collection Tool and Data Collection Process

This study is a part of the master's thesis research conducted by the second author. The items presented in Table 1 were directed to the teachers in an open-ended questionnaire format. In order to determine the teachers' general understanding of the open-ended question, the first item was posed, and the teachers were also asked to provide an example. In the second item, the teachers' conceptions of the mathematical open-ended question were intended to be revealed, and they were asked to present an example again. To obtain more insights into the teachers' understanding of the mathematical open-ended question, the last question was posed to see whether they would present different examples when they thought about the issue again.

Table1. Data Collection Tool Questions

Item No.	Item
1	What do you think an open-ended question is? Explain by giving an example.
2	What do you think a mathematical open-ended question is? Explain by giving an example.
3	Write down all possible examples of mathematical open-ended questions that come to mind other than the ones above.

A pilot study was conducted with 10 teachers to see whether this questionnaire served its purpose. It revealed that the questionnaire's items served their purpose that the teachers found the items understandable, and as a result, the main study was conducted with 40 middle school mathematics teachers. The data collection process was carried out on the Zoom platform with 34 teachers due to the COVID-19 pandemic conditions. The questionnaire was administered to the teachers one by one in the Zoom environment, and their responses were written on the shared screen. During this process, the researcher only wrote down the given responses, and did not interfere. The Zoom interviews lasted about 30 minutes. Data were obtained face-to-face with 6 of the teachers and again no direction or intervention was made to the participants.

Data Analysis

Data analysis was carried out in three stages. In the first stage, the responses of the teachers about the definition of the open-ended question were simultaneously subjected to content analysis by two researchers. As shown in Table 2, we constructed a data analysis framework for examining how teachers defined open-ended questions by using their responses for the first two items of the questionnaire. Examples from teachers' responses for each code are given, and since the codes are self-explanatory, their descriptions are not included.

For data analysis, firstly the codes were determined, then the common codes were brought together to form four categories. Therefore, data analysis was carried out with an inductive and exploratory approach (Azungah, 2018). Moreover, original categories and a novel data analysis framework specific to this study were produced. In this framework, there are some indications defining the open-ended question, such as if the appearance of the question was mentioned, teachers' responses were put under the 'form' category; if the result or output of the question was mentioned, the responses were put under the 'outcome' category; if the process or solution method of the question was mentioned, the responses were put under the 'process' category; and lastly, if the functionality of the question was mentioned, the responses were put under the 'functionality' category. For cases where the question was left unanswered, the 'unanswered' category and the answers that could not be included in the previous categories were put under the 'others' category. In total, six main categories are constructed to analyze the related data.

Table 2. Data Analysis Framework for the Definition of Open-Ended Question

Category	Code	Example
Form	Non-multiple-choice questions	Teacher (T)15: Questions without choices.
	Classic questions	Teacher (T)30: Classic questions.

Outcome	Questions allowing for a different interpretation	T28: These are questions that are left to the initiative of the student and given the right to interpret.
	Questions with multiple correct answers	T21: Questions with more than one answer are called open-ended.
	Questions with uncertain answers	T6: These are questions whose answers vary according to the student, and the answers are not ultimately certain.
	Questions with unstructured answers	T37: There are questions that are answered in a few sentences or text.
Process	Questions allowing for different solution methods	T2: Mathematical questions with different solution methods are called open-ended.
Functionality	Questions providing details about student thinking and knowledge	T17: A method of asking questions providing in-depth access to students' conceptions about a subject without limitations.
	Questions requiring reasoning	T11: Types of questions requiring reasoning
	Questions requiring higher-order thinking skills	T10: It allows students to think high-levelly, such as guessing and reasoning.
	Questions requiring proof	T8: Requiring proof.
Unanswered		-
Others		T8: Unproven mathematics problems.

In the second stage, the examples from the teachers were analyzed. The aim here was to determine whether the questions presented by the teachers for the open-ended question were really open-ended. For this aim, the analysis framework presented in Table 3 was taken from the relevant literature, as it was appropriate for the analysis of the collected data (Bingölbali & Bingölbali, 2020). The data analysis was carried out with a deductive approach (Azungah, 2018) using a ready-made analysis framework. This framework consists of two main categories: closed-ended and open-ended questions. The framework that we used for the data analysis includes a total of four categories, together with the category of 'unanswered' and 'others'. In this framework, questions with a single correct answer and definite correct answers are considered closed-ended, and questions with variable correct answers and infinitely correct answers are considered open-ended.

Table 3. Data Analysis Framework for Analyzing Teacher-Presented Examples

Category	Codes	Explanation	Example
Closed-ended questions	Questions with one correct answer	Questions have only one correct answer.	T29:1/6+1/3=? Find it.
	Questions with definite correct answers	Questions have more than one correct answer, but the number of answers is fixed.	T7: What are the integer divisors of 60? Write them all down.
Open-ended questions	Questions with variable correct answers	Questions have more than one answer, but may vary and differ from each other as the correct answer(s) will be chosen from a set of answers.	T19: Write down 5 homophones.
	Questions with infinitely correct answers	Questions have an infinite number of correct answers.	T22: Write down a problem and solve it.
Unanswered			-
Others			***T1:*** How reliable do you think the climate is?

The analysis made in the third stage was related to the example field to which the examples presented by the teachers belonged. The aim here was to determine whether the examples that the teachers presented were mathematical or not. As a result of the content analysis, 5 different categories were determined for the fields to which the examples belonged: (i) mathematical, (ii) mathematics education, (iii) philosophy of mathematics, (iv) non-mathematical, and (v) unanswered. The analysis framework that emerged based on this classification is presented in Table 4. The data were analyzed with an inductive approach (Azungah, 2018) for this part as well.

Table 4. Data Analysis Framework for Example Fields of Teachers' Examples

Category	Examples
Mathematical	T37: Give three examples of perfect square numbers less than 100.
Mathematics education	T8: What are the most important factors affecting mathematics education?
Philosophy of mathematics	T28: Is mathematics a discovery or an invention?

Non-mathematical	T2: What is the impact of COVID-19 on the economy?
Unanswered	

The data analysis process was carried out, including the construction of data analysis frameworks. This process made it possible to go through the data repeatedly and improve the analysis frameworks. After the analysis frameworks were constructed, the data were analyzed using these frameworks by the second researcher. The answers presented by each teacher were associated with the codes and labeled. Subsequently, both researchers analyzed all the data together. Therefore, a joint effort was made for the compatibility of all data with codes and categories. In addition, a common decision was reached on the classification of the data. Then, frequency tables were created for all data, and these findings were supported by qualitative quotations.

Precautions needed were taken to ensure the quality of the study (Shipman, 2014) both during the development of the data collection tool and during the data analysis process. For the validity of the data and the conclusions to be drawn from them, expert opinion was taken during the development of the data collection tool, and a pilot study was conducted. In order to strengthen the validity of the results obtained from the study, the quantitative data were supplemented with rich teacher citations. Data analysis was carried out by two different researchers at different times continuously, and this process contributed to both the reliability of the data analysis and the validity of the results of the study. In the data collection process, factors, such as being available and being motivated to participate in the study, were also taken into account in order to enable teachers to participate effectively, and in this way, sensitivity was shown for both collecting data securely and obtaining valid results.

RESULTS

Results section consists of three parts. In the first part, the findings about the teachers' conceptions of the open-ended question are presented. In the second part, the findings coming from the analysis of the teachers' personal examples of open-ended questions are provided, and in the last part, the findings related to the example field to which the examples belong are presented.

What Kind of Definitions Did the Teachers Provide for Open-Ended Questions?

The findings for the first two items of the questionnaire (the ones about the definition of open-ended questions) will be presented in this section. Prior to presenting the findings, it will be helpful for the reader to provide explanatory information presented in Table 5. The numbers given in the lines, such as 'form total' and 'outcome total' in the table, are the number of elements of the combination set of the codes under the categories. For example, four teachers referred to 'non-multiple-choice questions, seven teachers referred to 'classic questions' while defining the open-ended questions, and some of these teachers were the same, eight different teachers in total were found to refer to the form. In the table, '*n*' refers to the number of participants while '*r*' refers to the number of distinguished responses that all participants provide.

Table 5 shows that when teachers defined the open-ended question commonly, they gave 65 (=r) definitions in total, and 51 (=r) when asked to define mathematically. The findings show that 20% of the teachers made definitions for the open-ended question based on the 'form' of the question while giving a general definition. For a mathematical definition, 22.5% of the teacher referred to the 'form' of the question. Teachers described the open-ended question as 'the one with no choice and/or classical question' within the scope of the 'form' category. One teacher defined the open-ended question as "questions without choices" (T15), while another one expressed it as "classical questions" (T30). Some teachers hence utilized the appearance of the question to decide whether a question is open-ended. Some teachers also associated the open-ended question with the classical one and placed it at the opposite pole of 'yes-no' or 'multiple-choice' questions.

The findings also show that some teachers defined open-ended questions based on the number of answers they had or the form of answers. Table 5 shows that when asked to describe the open-ended question in a general way, the proportion of teachers who referred to the 'outcome' was 82.5% (33). Under the 'outcome' category, for a general definition, teachers made definitions by referring to these four codes: 'questions allowing for different interpretations' (32.5%), 'questions with multiple correct answers (25%), 'questions with uncertain answers' (22.5%) and 'questions with unstructured answers ' (25%). For example, one teacher defined an open-ended question as "questions that are open to interpretation, the answer of which can vary from person to person" (T22). Another teacher's conception of the open-ended question was as follows: "questions that do not have a single answer are called (open-ended questions)" (T12).

The responses presented for the mathematical definition of the open-ended question also showed similarities in Table 5. However, while the number of different teachers whose responses were considered within the scope of at least one code under the 'outcome' category was 33 for the general definition, this number was 23 for the mathematical definition. Under the 'outcome' category, in terms of distribution to the codes, the definitions made by the teachers for the mathematical open-ended question were as follows: 'allowing for different interpretations' (20%), 'questions with multiple correct answers (12.5%), 'questions with uncertain answers' (7.5%) and 'questions with unstructured answers' (22.5%). In this connection, for example, two teachers defined the open-ended question as follows: "it is the type of question that measures mathematics knowledge according to different interpretations" (T1), and "questions with more than one mathematical answer are called open-ended" (T7).

Table 5. Teachers' Definitions for Open-Ended Questions

Category	Code	General Frequency (n=40/r=65)	Mathematical Frequency (n=40/r=51)
Form	Non-multiple-choice questions	4(10%)	5(12.5%)
	Classic questions	7(17.5 %)	5(12.5%)
Form total		8 (20%)	9 (22.5%)
Outcome	Questions allowing for different interpretation	13 (32.5%)	8(20%)
	Questions with multiple correct answers	10 (25%)	5(12.5%)
	Questions with uncertain answers	9 (22.5%)	3(7.5%)
	Questions with unstructured answers	10(%25)	9(22.5%)
Outcome total		33 (82.5%)	23 (57.5%)
Process	Questions allowing for different solution methods	4 (10%)	7(17.5%)
Process total		4 (10%)	7(17.5%)
Functionality	Questions providing details about student thinking and knowledge	7(17.5 %)	-
	Questions requiring reasoning	1 (2.5%)	-
	Questions requiring higher-order thinking skills	-	9(22.5%)
	Questions requiring proof	-	2(5%)
Functionality total		8 (20%)	11 (27.5%)
Unanswered		-	3(7.5%)
Others		-	1(2.5%)

Table 5 also reveals that some teachers defined open-ended questions through the process, that is, the solution ways or methods. It shows that 10% of the teachers who defined the open-ended question commonly and 17.5 % of those who defined it mathematically stated that the questions with multiple solution methods were open-ended. On the other hand, some teachers defined the open-ended question over its functionality. Furthermore, Table 5 displays that 20% of the teachers who defined the open-ended question in general terms and 27.5% of those who defined it mathematically referred to the functionality of the open-ended question to define it. The responses of some teachers (17.5%) who defined the open-ended questions in general terms were considered under the code of ‘questions providing details about student thinking and knowledge'. These teachers referred to the functionality of the open-ended question rather than its definition. Furthermore, one participant expressed the open-ended question as one “requiring reasoning” (T11). When defining it mathematically, some teachers defined open-ended questions as ‘questions requiring high-level thinking skills’ (22.5%) and ‘questions requiring proof’ (5%). One teacher explained the open-ended question as “it is the type of question that develops high-level thinking skills” (T10), and hence defined it through its function. Further, the findings also show that when the definition of an open-ended question was asked in general terms, all teachers answered the question, but when asked in mathematical terms, 3 teachers left the question unanswered

What Kind of Examples Did the Teachers Provide for the Open-Ended Questions?

Before presenting the findings, it will be helpful for the reader to provide explanatory information about Table 6 as it contains the findings. The values given in the 'total' rows correspond to the number of different teachers whose answers were placed under the relevant category. In the table, ‘*n*’ represents the number of participants and ‘*e*’ represents the number of examples presented by teachers. It should be noted that since some teachers' answers include examples of both open-ended and closed-ended questions, the total percentages in the table exceed 100%. The numbers in the ‘total’ lines refer to the numbers of different teachers for the related category.

Table 6 shows that 22.5% of the examples presented by the teachers who defined the open-ended question in general terms were closed-ended, 67.5% were open-ended, and 2.5% were in the other category. 15% (6) of the teachers did not provide an example. Therefore, 32.5% of the teachers did not give an example of open-ended questions. Some teachers presented closed-ended questions with one correct answer (12.5%) and certain correct answers (12.5%) as examples for open-ended questions. For example, a teacher gave an example of a closed-ended question with a certain answer, such as “What are the integer divisors of 60? Write them all down” (T7), as an example for the open-ended question. Another teacher provided the example of "1/6+1/3=? Find it out” (T29), which has one correct answer as an example for the open-ended question.

The findings show that 67.5% of the examples given by the teachers who presented the open-ended question example correctly had variable correct answers. For example, a teacher stated that “What I am answering right now is 'What do you think is an open-ended question? Explain by giving an example is an open-ended question” (T38). Another teacher presented an example as "Write five homophones" (T19). There was no teacher who presented an example of open-ended questions with infinite correct answers.

Table 6. Analysis of Examples Presented By Teachers for Open-Ended Questions

Category	Codes	**General (n=40/ e=38)**	**Mathematical (n=40/e=35)**	**Other mathematical (n=40/e=34)**
Closed-ended questions	Questions with one correct answer	5(12.5%)	17 (42.5%)	17 (42.5 %)
	Questions with definite correct answers	5 (12.5%)	4 (10%)	3 (7.5 %)
Total		9 (22.5%)	17 (42.5%)	11 (27.5%)
Open-ended questions	Questions with variable correct answers	27 (67.5%)	12 (30%)	9 (22.5%)
	Questions with infinitely correct answers	0 (0%)	1 (2.5 %)	0 (0%)
Total		27 (67.5%)	13 (32.5%)	7 (17.5%)
Unanswered		6 (15%)	10(25%)	21 (52.5 %)
Others		1 (2.5%)	1 (2.5%)	5 (12.5%)

The analysis of the examples presented for the first item (one for mathematical open-ended questions) showed that 42.5% of the teachers presented closed-ended questions as open-ended questions. For example, the example of "How many times 2 is half of 44?" (T1) with one correct answer and the example of "What are the divisors of the number 60? Write all of them" (T7) with certain correct answers were given by the teachers considering that they were open-ended questions. Table 6 also shows that the rate of teachers who gave mathematically correct open-ended questions decreased to 32.5% and 25% of the teachers left the item unanswered. An open-ended question with infinitely correct answers was given by only one teacher: "Write down a problem and solve it" (T22). Table 6 also shows that 30% of the teachers gave the example of open-ended questions with variable correct answers. For example, a teacher presented an example with a variable answer as "Give three examples of perfect square numbers less than 100" (T37). Regarding the open-ended question, another teacher replied, "We cannot think of an example because we did not ask it" (T2). Similarly, another teacher stated that "I cannot think of it. You know what the subject is, but it's hard to come up with an idea when you are asked to give one. I am currently experiencing this situation" (T37).

Table 6 also presents the example analyses for the third item of the questionnaire. The responses given to this item also revealed a similar pattern, and 52.5% of the teachers could not provide a new example. The absence of new examples for this item indicates that teachers' understandings of the open-ended question example were largely limited to the examples they provided for mathematical examples. Table 6 shows that more responses were categorized under the ‘others’ category for this item, and the example of "It looks like it will be very hot today" (T13) was such one.

In some cases, both closed and open-ended questions were encountered in the answers given by the same teacher. For instance, one teacher (T9) provided these two examples for open-ended questions: (i) “Ali was 8 years old when his brother was born. What is the age difference between the two brothers after 10 years?” and (ii) "Write 7 prime numbers less than 100 and greater than 40". While the first one is closed-ended as it has one correct answer, the second one is open-ended. Another teacher (T13) also provided contradictory examples such as "Draw a rectangle whose area is 24 unit squares" and "Questions about operation priority: 16-4:1+5x3=?". These responses show the lack of understanding of the teachers regarding open-ended questions at the conceptual level. In addition to this, the teacher (T13) made a statement such as this as well: “I really think I have misconceptions about open-ended questions”.

Which Fields Did the Open-Ended Questions Presented by the Teachers Belong to?

The examples given by the participants were analyzed concerning the five categories presented in Table 7 to determine the fields to which they belonged. The findings are presented based on the number of participants (teachers) and the number of examples (answers). For instance, while 12.5% of the teachers gave mathematical examples (line-1), these examples were 13% of the presented examples (line-1). Table 7 shows that 65% of the teachers gave non-mathematical examples, while only 12.5% (5 teachers) provided mathematical examples when an example was requested along with the first item. For example, one teacher presented a non-mathematical example, such as “What is the impact of COVID-19 on the economy?” (T2). Some of the teachers' answers were from the field of mathematics education, such as "What are the most important factors affecting mathematics education?" (T8), and from the field of Philosophy of Mathematics, such as "Is mathematics a discovery or an invention" (T28). The findings also showed that six teachers did not provide any examples.

Table 7. The Fields of the Examples Presented By the Teachers for the Open-Ended Question

	General		Mathematical		Other Mathematical	
Category	**Teacher (n=40)**	**Response (e=38)**	**Teacher (n=40)**	**Response (e=35)**	**Teacher (n=40)**	**Response (e=34)**

Mathematical	5 (12.5%)	5(13%)	26 (65%)	31(88.5%)	18 (45%)	31 (91%)
Mathematics education	2 (5%)	2 (5%)	1 (2.5%)	1 (3%)	-	
Philosophy of mathematics	1 (2.5%)	1 (3%)	3(7.5%)	3(8.5%)	-	-
Non-mathematical	26 (65%)	30 (79%)	-	-	3 (7.5%)	3 (9%)
Unanswered	6 (15%)		10 (25%)		21 (52.5%)	

When directly asked for a mathematical example, this time most of the teachers (65%) provided mathematical ones. However, in this case, the number of teachers who could not provide any examples increased (25%). When other mathematical examples were requested from the teachers, the rate of unanswered increased to 52.5%. All these findings show that teachers have difficulties in presenting mathematical examples for open-ended questions.

DISCUSSION

The findings show that teachers conceptualize open-ended questions differently, even some have profound misconceptions about open-ended questions. The teachers mostly defined open-ended questions in general and mathematically through their 'outcome'. Some other teachers defined the open-ended question in terms of 'form' and interpreted the open-ended question as a 'non-multiple-choice' and/or 'classical' one. Instead of defining the open-ended question directly, the teachers explained what it was not. While defining the open-ended question, some teachers took the process as a basis in which they described the questions that can be solved through different solutions as open-ended questions. Functionality was another category that we used to describe how the teacher defined the open-ended question, and this category came to the foreground for the mathematical one. Open-ended questions were defined, for instance, as 'questions requiring high-level thinking skills' and the 'question requiring proof', and hence the functionality of the questions were brought up the forward rather than its definition.

It appears that the teachers' conceptions of the open-ended question overlapped in some dimensions with the expert definitions given in the literature. For example, open-ended questions are considered ones that allow different interpretations, answers, or outputs (Aziza, 2021; Silver, 1995). Determining whether the question was open-ended through its output was the most frequently applied criterion by the participants of this study. In this connection, 'allowing different interpretations', 'having multiple correct answers', 'uncertain answers', and 'unstructured answers ' were the codes containing the teachers' answers. However, as can be seen in the findings section, the definitions made were not based on the number of answers, as Bingölbali & Bingölbali (2020) stated. The teachers' definitions were quite inclusive and general.

Open-ended questions are associated with the terms "constructed response" and "extended response" in the field of measurement-evaluation (Hogan & Murphy, 2007; İnceçam et al., 2018; McMillan, 2017). Here again, the main emphasis is on the output. The teachers also emphasized this aspect of open-ended questions by making definitions that could be included in the scope of 'structuring the response'. They conceptualized the open-ended question as one with 'no multiple-choice' and 'classical', and hence made a definition based on the form of the question as well. The lack of 'multiple-choices' offers students the opportunity to construct their answers, and hence the teachers defined the open-ended question through this term.

The relevant literature shows that questions that allow different solution methods are also described as open-ended questions (Aziza, 2021; Silver, 1995). However, Bingölbali & Bingölbali (2020) stated that if different solution methods are requested as output in a question, then the problem can be evaluated as closed or open-ended based on the number of outputs. Therefore, whether the problem is open-ended or not is related to whether a different solution method is directly requested, rather than the possibility of different solution methods. The findings showed that the participants of this study regarded the 'questions that allow for different solution methods', which we considered within the scope of the process category, as open-ended. Teachers brought different solution methods to the agenda, especially in defining the mathematical open-ended problem.

In defining open-ended questions, teachers also referred to the functionality of the question. The teachers defined the questions that "provide details about student thinking and knowledge" and "require high-level thinking skills" as open-ended ones. Functionality is essentially about the function of the question, not the definition. The teachers did not provide a definition here but attempted to make sense of the open-ended question through its function. Functionality is a category that does not coincide with the experts' definitions presented in the relevant literature, which means that some teachers understand the open-ended question that is far from the experts' understanding of the open-ended question.

Pehkonen (1997, p. 8) defined the open-ended problem in terms of problem situations (e.g., investigations, problem posing) requiring active participation from the students. It appears that Pehkonen (1997) made a definition for the open-ended problem based on the types (features) of tasks that require the use of high-level thinking skills, rather than the responses to the problem or the process it requires. Some teachers also referred to form for the open-ended question, but instead of emphasizing what Pehkonen mentioned, they stated that the question is not multiple-choice or that the question is classic. There was only one teacher (T22) gave an example of a problem-posing question ("write down a problem and solve it") with infinitely correct answers. On the other hand, the tasks that Pehkonen presented for the open-ended problem require the use of higher-order thinking skills. In other words, they are related to the teacher's responses falling under the functionality category in this study. For the mathematical open-ended question, the teachers defined open-ended question as one "requiring high-level thinking skills", but they did not specifically name the tasks that Pehkonen explicitly stated.

Reitman (1966) divided a problem into three parts (the given, operation, and outcome) and stated that when at least one of these is not given, then the problem is open-ended. This definition of Reitman is frequently used for problem-solving, especially by practitioners. In this study, the teachers did not define open-ended questions specifically through Reitman's terms and did not provide examples compatible with this definition.

While the definitions made by the teachers overlap with the ones presented in the literature, one may wonder if the examples they presented are actually open-ended. The findings show that only 67.5% of the examples given for the general open-ended questions were open-ended, and 32.5% of the examples were not open-ended. None of the presented open-ended questions had infinitive correct answers. When a mathematical example was requested, 42.5% of the teachers provided closed-ended questions as open-ended questions. The rate of teachers who gave the correct example of open-ended questions was only 32.5%. It is noteworthy that the rate of those who left the question ‘unanswered’ is high when a mathematical example is requested.

Although the teachers define an open-ended question, the examples of open-ended questions actually show that they have significant difficulties with what an open-ended question is. Examples are known for their contributions to the understanding and interpretation of definitions in mathematics learning-teaching (Watson & Mason, 2002). Examples are also considered to provide insight into the student's understanding and mathematical knowledge (Liz et al., 2006). In this study, the requested examples provided insights into the teachers' understanding of (difficulties with) the open-ended question. The main finding was that most teachers had difficulty giving examples of mathematical open-ended questions. This finding is in parallel with Pehkonen (1999), which found that about half of the participating teachers did not provide an appropriate definition for an open-ended task and that only a quarter of the participants knew what an open-ended task was.

As far as example fields to which the examples belong are concerned, it is interesting that mathematics teachers mostly think of non-mathematical examples. They were able to give mathematical examples only when the mathematical ones were directly requested. However, when a mathematical example was directly requested, mathematics teachers had more difficulties giving one. It appears that mathematics teachers' images of open-ended examples (Tall & Vinner, 1981) mostly belong to non-mathematical fields.

CONCLUSION

This study shows that mathematics teachers define open-ended questions in terms of outcome/result, process, form, and functionality. It is observed that teachers have difficulties explaining open-ended questions, especially in mathematical terms. In addition, the examples presented as open-ended reveal that some teachers regard closed-ended questions as open-ended questions and conceptualize the open-ended questions incorrectly. These findings show that teachers lack a conceptual perspective, which can help them make sense of open-ended questions and distinguish them from closed-ended ones. At this point, the analytical frameworks used in this study can provide teachers with a conceptual perspective on open-ended questions and also can help them develop open-ended questions. Considering that the use of open-ended questions is constantly on the agenda for mathematics learning and teaching, more conceptual studies are required not only to enrich teachers' understanding of the open-ended questions but also to sharpen their open-ended question judgments.

ACKNOWLEDGEMENT

This paper stems from the ongoing Master's thesis of the second author.

REFERENCES

Aziza, M. (2021). A Teacher Questioning Activity: The Use of Oral Open-ended Questions in Mathematics Classroom. *Qualitative Research in Education*, *10*(1), 31–61. https://doi.org/10.17583/qre.2021.6475

Azungah, T. (2018). Qualitative research: deductive and inductive approaches to data analysis. *Qualitative Research Journal*, *18*(4), 383–400. https://doi.org/10.1108/QRJ-D-18-00035

Becker, J. P., & Shimada, S. (1997). *The Open Ended Approach*. National Council of Teachers of Mathematics.

Bennevall, M. (2016). *Cultivating Creativity in the Mathematics Classroom using Open-ended Tasks : A Systematic Review TT - Utvecklande av kreativitet i matematikklassrum med hjälp av öppna problem : En systematisk genomgång A Systematic Review (swe): Vol. Independen*. http://liu.diva-portal.org/smash/get/diva2:909145/FULLTEXT01.pdf

Bingolbali, E. (2020a). An analysis of questions with multiple solution methods and multiple outcomes in mathematics textbooks. *International Journal of Mathematical Education in Science and Technology*, *51*(5), 669–687. https://doi.org/10.1080/0020739X.2019.1606949

Bingolbali, E. (2020b). Çok Doğru Cevaplı ve Çok Çözüm Metotlu Etkinliklerin Ortaokul Matematik Ders Kitaplarındaki Yeri [An Examination of Tasks in Elementary School Mathematics Textbooks in Terms of Multiple Outcomes and Multiple Solution Methods]. *International Journal of Educational Studies in Mathematics*, *7*(4), 214–235.

https://doi.org/10.1080/0020739X.2019.1606949

Bingölbali, E., & Bingölbali, F. (2020). Divergent Thinking and Convergent Thinking: Are They Promoted in Mathematics Textbooks? *International Journal of Contemporary Educational Research*. https://doi.org/10.33200/ijcer.689555

Bingölbali, E., & Bingölbali, F. (2021). An Examination of Open-Ended Mathematics Questions' Affordances. *International Journal of Progressive Education*, *17*(4), 1–16. https://doi.org/10.29329/ijpe.2021.366.1

Boulton-Lewis, G. M., Smith, D. J. H., McCrindle, A. R., Burnett, P. C., & Campbell, K. J. (2001). Secondary teachers' conceptions of teaching and learning. *Learning and Instruction*, *11*(1), 35–51. https://doi.org/10.1016/S0959-4752(00)00014-1

Bragg, L., & Nicol, C. (2008). Designing open-ended problems to challenge preservice teachers' views on mathematics and pedagogy. *PME 32: Mathematical Ideas: History, Education and Cognition: Proceedings of the 32nd Conference of the International Group for the Psychology of Mathematics Education*, 201–208.

Cai, J. (1995). A Cognitive Analysis of U. S. and Chinese Students' Mathematical Performance on Tasks Involving Computation, Simple Problem Solving, and Complex Problem Solving. *Journal for Research in Mathematics Education. Monograph*, *7*, i–151. https://doi.org/10.2307/749940

Cai, J. (2000). Mathematical Thinking Involved in U.S. and Chinese Students' Solving of Process-Constrained and Process-Open Problems. *Mathematical Thinking and Learning*, *2*(4), 309–340. https://doi.org/10.1207/S15327833MTL0204_4

Çekmez, E., Yildiz, C., & Bütüner, S. Ö. (2012). Fenomenografik Araştırma Yöntemi [Phenomenographic Research Method]. *Necatibey Faculty of Education, Electronic Journal of Science and Mathematics Education*, 6(2), 77.

Clarke, D. J., Sullivan, P., & Spandel, U. (1992). *Student response characteristics to open-ended tasks in mathematical and other academic contexts* (P. Sullivan, U. Spandel, & Australian Catholic University. Mathematics Teaching and Learning Centre (eds.)). The Centre.

Gracin, G. D. (2018). Requirements in mathematics textbooks: a five-dimensional analysis of textbook exercises and examples. *International Journal of Mathematical Education in Science and Technology*, *49*(7), 1003–1024. https://doi.org/10.1080/0020739X.2018.1431849

Han, S., Rosli, R., Capraro, R. M., & Capraro, M. M. (2011). The Textbook Analysis on Probability: The Case of Korea, Malaysia and U.S. Textbooks. *Journal of the Korean Society of Mathematical Education Series*, *15*(2), 127–140.

Hogan, T. P., & Murphy, G. (2007). Recommendations for Preparing and Scoring Constructed-Response Items: What the Experts Say. *Applied Measurement in Education*, *20*(4), 427–441. https://doi.org/10.1080/08957340701580736

İnceçam, B., Demir, E., & Demir, E. (2018). Ortaokul Öğretmenlerinin Sınıf İçi Ölçme ve Değerlendirmelerde Yazılı Yoklamalarda Kullandıkları Açık Uçlu Maddeleri Hazırlama Yeterlikleri [Competencies of Middle School Teachers to Prepare Open-Ended Items Used in Open-Ended Test for In-Classroom Assess. *İlköğretim Online*, *17*(4), 1912–1927. https://doi.org/10.17051/ilkonline.2019.506900

Kasar, N. (2013). *Matematik Derslerinde Alternatif Çözüm Yollarına ve Farklı Soru Türlerine Ne Ölçüde Yer Verilmektedir?: Sınıf İçi Uygulamalardan Örnekler [To What Extent Alternative Solution Methods and Different Question Types Are Given Place in Mathematics Teaching?: E*. University of Gaziantep.

Klavir, R., & Hershkovitz, S. (2008). Teaching and Evaluating 'Open-Ended' Problems. *International Journal for Mathematics Teaching and Learning*, *20*(5), 1–24.

https://www.cimt.org.uk/journal/klavir.pdf

Kwon, O. N., Park, J. H., & Park, J. S. (2006). Cultivating divergent thinking in mathematics through an open-ended approach. *Asia Pacific Education Review*, *7*(1), 51–61. https://doi.org/10.1007/BF03036784

Leung, S. S. (1997). On the open-ended nature in mathematical problem posing on the open-ended nature in mathematical problem posing. In E. Pehkonen (Ed.), *Use of Open-ended Problems in Mathematics Classroom* (pp. 26–33). Department of Teacher Education, University of Helsinki. http://coreylee.me/en/publications/2001_self-efficacy_change.pdf%5Cnhttp://files.eric.ed.gov/fulltext/ED419714.pdf

Lin, C.-Y., Becker, J., Ko, Y.-Y., & Byun, M.-R. (2013). Enhancing pre-service teachers' fraction knowledge through open approach instruction. *The Journal of Mathematical Behavior*, *32*(3), 309–330. https://doi.org/10.1016/j.jmathb.2013.03.004

Liz, Bills, Dreyfus, T., Mason, J., Tsamir, P., Watson, A., & Zaslavsky, O. (2006). Exemplification in mathematics education. *30th Conference of the International Group for the Psychology of Mathematics Education*, *Leinhardt 2001*, 126–154. http://users.mct.open.ac.uk/jhm3/PME30RF/PME30RFPaper.pdf

McMillan, J. H. (2017). *Classroom Assessment: Principles and Practice that Enhance Student Learning and Motivation*. Pearson Education.

Merriam, S. B., & Tisdell, E. J. (2016). *Qualitative Research: A Guide to Design and Implementation*. John Wiley & Sons.

MoNE. (2017). *8. Sınıf Merkezî Ortak Sınavlar Matematik Dersi Açık Uçlu Sorular Yapılandırılmış Cevapanahtarı Örnekleri [Open-Ended Questions for Eight-Grade Central Common Mathematics Exams and the Answer Key Examples]*. http://odsgm.meb.gov.tr/meb_iys_dosyalar/2017_09/15135732_Mat_acik_uclu.pdf.

Nieminen, J. H., Chan, M. C. E., & Clarke, D. (2022). What affordances do open-ended real-life tasks offer for sharing student agency in collaborative problem-solving? *Educational Studies in Mathematics*, *109*(1), 115–136. https://doi.org/10.1007/s10649-021-10074-9

Nohda, N. (2000). Teaching by open-approach method in Japanese mathematics classroom. *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, *1*, 39–53.

OECD. (2017). PISA 2015 (Assessment and Analytical Framework : Science, Reading, Mathematic, Financial Literacy and Collaborative Problem Solving). In *Ming Pao Daily News*. OECD Publishing.

ÖSYM. (2017). *ÖSYS: Öğrenci Seçme ve Yerleştirme Sistemi [ÖSYS: Student Selection and Placement System]*. https://www.osym.gov.tr/TR,12909/2017-lisans-yerlestirme-sinavlari-2017-lys-acik-uclu-sorular-hakkinda-bilgilendirme-ve-acik-uclu-soru-ornekleri-05012017.html

Pehkonen, E. (1997). Introduction to the concept "open-ended problem." In E. Pehkonen (Ed.), *Use of Open-ended Problems in Mathematics Classroom* (pp. 7–11). Department of Teacher Education, University of Helsinki.

Pehkonen, E. (1999). In-service teachers' conceptions on open tasks. In G. N. Philippou (Ed.), *MAVI-8 Proceedings, Research on Mathematical Beliefs* (pp. 87–95). University of Cyprus.

Reitman, W. R. (1966). *Cognition and Thought: An Information Processing Approach*. Wiley.

Shipman, M. D. (2014). *The Limitations of Social Research*. Taylor & Francis.

Silver, E. A. (1992). Assessment and mathematics education reform in the united states. *International Journal of Educational Research*, *17*(5), 489–502. https://doi.org/10.1016/S0883-

0355(05)80007-2

Silver, E. A. (1995). The Nature and Use of Open Problems in Mathematics Education: Mathematical and Pedagogical Perspectives. *Zentralblatt Fur Didaktik Der Mathematik/International Reviews on Mathematical Education*, *27*(2), 67–72.

Sullivan, P., & Clarke, D. (1992). Problem solving with conventional mathematics content: Responses of pupils to open mathematical tasks. *Mathematics Education Research Journal*, *4*(1), 42–60. https://doi.org/10.1007/BF03217231

Sullivan, P., Warren, E., & White, P. (2000). Students' responses to content specific open-ended mathematical tasks. *Mathematics Education Research Journal*, *12*(1), 2–17. https://doi.org/10.1007/BF03217071

Sullivan, P., Warren, E., White, P., & Suwarsono, S. (1998). Different forms of mathematical questions for different purposes: Comparing student responses to similar closed and open-ended questions. In C. Kanes, E. Warren, & M. Goos (Eds.), *Teaching Mathematics in New Times*. MERGA.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*(2), 151–169. https://doi.org/10.1007/BF00305619

Watson, A., & Mason, J. (2002). Extending example spaces as a learning/teaching strategy in mathematics. *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education*, *4*, 4–377.

Yang, D. C., Tseng, Y. K., & Wang, T. L. (2017). A comparison of geometry problems in middle-grade mathematics textbooks from Taiwan, Singapore, Finland, and the United States. *Eurasia Journal of Mathematics, Science and Technology Education*, *13*(7), 2841–2857. https://doi.org/10.12973/eurasia.2017.00721a

Yıldırım, A., & Şimşek, H. (2011). *Sosyal bilimlerde nitel araştırma yöntemleri [Qualitative research methods in the social sciences]*. Seçkin.

Zaslavsky, O. (1995). Open-ended tasks as a trigger for mathematics teachers' professional development. *For the Learning of Mathematics*, *15*(3), 15–20. http://www.jstor.org/stable/40248183

Zhu, Y., & Fan, L. (2006). Focus on the Representation of Problem Types in Intended Curriculum: A Comparison of Selected Mathematics Textbooks from Mainland China and the United States. *International Journal of Science and Mathematics Education*, *4*(4), 609–626. https://doi.org/10.1007/s10763-006-9036-9