

# High School Students' Multiple Representation Translation Skills on One-Dimensional Motion: A Cross-Grade Study

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**ABSTRACT** Multiple representations are widely recognized for their significant role in concept learning. This study aimed to investigate the multiple representation translation skills of high school students at different grade levels about the concept of one-dimensional motion. 239 9th, 10th, and 11th-grade students participated in the study using a developmental research model. The data collection tool consisted of questions that required translating figures, tables, graphs, verbal explanations, and algebraic representations into other representation types in a multiple-representation translation test focusing on one-dimensional motion. Data analysis involved evaluating the translation among representations for each category and analyzing the multiple representation translation skills across different grade levels using one-way analysis of variance (ANOVA). The results revealed that students successfully translated from figure, table, and graphical representations to other forms while encountering challenges in translation from verbal and algebraic representations. Furthermore, the ANOVA results indicated a significant difference between the 9th and 11th grades, favoring the 11th grade.

**Keywords** Multiple representations, One-dimensional motion, Translating among representations, Cross-grade.

## 1. INTRODUCTION

Today, education is shifting away from rote memorization, calculation-based problems, and traditional paper-and-pencil tests towards a learning approach that emphasizes analytical abilities, problem-solving autonomy, and deep conceptual understanding (Erbaş, 2005). One effective method to facilitate this type of learning is the utilization of multiple representations, which is recommended by the National Council of Teachers of Mathematics (NCTM) to enhance learning in education (NCTM, 2000). Multiple representations enable students to engage in a problem-solving, causal, and communicative learning environment, different from the conventional approach that focuses on static knowledge and isolated skills unrelated to other subjects and daily life (Erbaş, 2005).

To comprehend the concept of multiple representations, it is advisable first to examine the definition of representation. Representations can be characterized as systems operating at either the traditional or cultural level, exhibiting a dynamic and non-fixed nature that allows for personal and idiosyncratic interpretations (Goldin & Kaput, 1996). Multiple representations involve the ability to express a concept using diverse forms of

representation and establish connections between them (NCTM, 2000). Based on this definition, the classification of multiple representations can be explored. Researchers have categorized representation styles into different groups. Herbel-Eisenmann (2002) classified them into four categories: graphics, tables, equations, and problem situations. Cleaves (2008) identified six classes of multiple representations: numerical (tables), graphical, pictorial, verbal, symbolic (equations), and physical.

Moreover, internal representations and external representations have also been classified. Internal representations refer to the cognitive building blocks that reflect students' mental models (Rau, 2017), while external representations encompass tangible forms such as graphs, tables, or equations (Ainley, Barton, Jones, Pfannkuch & Thomas, 2002). Internal representations are symbolic constructs formed within the mind through cognitive processes, whereas external representations manifest as observable situations and symbols resulting from the conversion of internal representations into behavior (Goldin & Kaput, 1996).

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The ability to express a concept using different representations and establish connections between them is recognized as a crucial skill (NCTM, 2000). It is also acknowledged that individuals who can successfully translate among different representations enhance their conceptual understanding (Işık, Işık, & Kar, 2011). Consequently, students attain a higher level of knowledge, improve their associative skills, and enhance their conceptual comprehension (İncikabı & Biber, 2018). By incorporating multiple representations in problem-solving activities, students experience improved perceptual development, better symbol interpretation, and enhanced verbal abilities, leading to a more profound understanding of mathematics (Van Heuvelen & Zou, 2001). Additionally, multiple representations demonstrate to students that problems have multiple solutions and can be approached from various angles, facilitating problem-solving processes (Dufrense, Gerace & Leonard, 1997). Furthermore, multiple representations offer opportunities to present concepts and relationships through equations, formulas, graphs, tables, figures, and symbols (Çelik & Sağlam-Arslan, 2012).

Furthermore, multiple representations contribute to verbal comprehension by utilizing words and sentences in the problem statement, enhancing numerical understanding through tables, promoting visual perception through graphics, and facilitate algebraic learning by incorporating symbols (İncikabı & Biber, 2018). These contributions enhance problem-solving abilities (Van Heuvelen & Zou, 2001). Experts in the field employ multiple representation skills when examining problems from different perspectives, whereas novices focus on formulating the appropriate mathematical equation and finding the solution (Kohl, Rosengrand, & Finkelstein, 2007). While this distinction may be less noticeable when solving simple problems, it becomes particularly significant when tackling more challenging problems (Kohl, Rosengrand & Finkelstein, 2007). Individuals who rely on a single representation while engrossed in a problem cannot gain a comprehensive perspective.

In contrast, using multiple representations enables examining problems from multiple aspects and perspectives (McGowan & Tall, 2001). Consequently, this ability to employ multiple representations distinguishes experts from novices in problem-solving. To cultivate these skills, students are encouraged to utilize multiple representations, articulate the problem, and foster a more profound understanding (NCTM, 2000).

Using multiple representations plays a pivotal role in enhancing success across various domains. Specifically, students who employ multiple representations when solving mathematical problems find solutions more efficiently and witness an increase in their overall success (Dreher & Kuntze, 2015). Moreover, such students exhibit advancements in conceptual understanding and the

development of cognitive structures (Abdurrahman, Setyaningsih & Jarmo, 2019; Baptista, Martins, Conceição, & Reis, 2019). The incorporation of multiple representations aids in anticipating the types of questions encountered in assessments like TIMMS and PISA, contributing to successful outcomes (Abdurrahman, Setyaningsih & Jarmo, 2019). Despite these benefits, multiple representations often do not receive adequate attention in educational settings, resulting in students possessing lower levels of proficiency in this skill (Lusiyana, 2019; Pebriana, Supahar, Pradana & Mundilarto, 2021). However, there is an optimistic attitude, as proficiency in multiple representations is believed to develop with age (Ainsworth, 2006). This could be attributed to the notion that children tend to think more simplistically than adults when dealing with multiple representations (Siegler & Opfer, 2003). Analyzing the development of various representation types at the high school level allows for the evaluation of changes in proficiency over subsequent years.

Multiple representations play a crucial role in physics education (Pebriana, Supahar, Pradana & Mundilarto, 2022). Physics educators widely acknowledge the necessity of incorporating multiple representations into lessons due to their significant contributions to teaching and learning (Klein, Müller, & Kuhn, 2017; Kohl, Rosengrand, & Finkelstein, 2007; Van Heuvelen & Zou, 2001). Therefore, in physics teaching, it is essential to go beyond solely relying on numerical expressions and include practices that involve multiple representations to facilitate conceptual understanding and promote deep learning (Umrotul, Jewaru, Kusairi & Pramono, 2022). Kinematics, in particular, is a suitable domain for assessing students' proficiency in multiple representations and their ability to translate among different representations, as it often reveals learning difficulties related to multiple representations (Klein, Müller, & Kuhn, 2017). To comprehend kinematics concepts effectively, students must first understand and engage with multiple representations (Umrotul, Jewaru, Kusairi & Pramono, 2022). This necessitates their recognition and familiarity with various types of representations. Otherwise, students may solely rely on equations in problem-solving situations, leading to difficulties in reaching correct solutions due to reliance on memorization rather than deep comprehension (Umrotul, Jewaru, Kusairi & Pramono, 2022). Examining students' representation skills in subjects that require proficiency in multiple representations, such as kinematics, can discern the types of knowledge they possess, including both memorized knowledge and expert knowledge in the field (Kohl, Rosengrand & Finkelstein, 2007). Linear motion is the foundation for other kinematics topics, and individuals who grasp this subject are better equipped to solve questions in related areas (Kusairi, Noviandari, & Pratiwi, 2019). When students approach and explain kinematics concepts using multiple representations, such as

graphics and verbal expressions, they find it easier to solve problems, thereby enhancing their problem-solving skills (Saputra, Jumadi, Paramitha & Sarah, 2019). Without a deep understanding of the subject, students may struggle to comprehend the content and resort to solving questions solely through memorized mathematical operations (Yener & Güzel, 2010).

An examination of research in this field reveals several categories, including students' ability to utilize multiple representations, the interest and competence of teachers and pre-service teachers in using multiple representations, and the impact of employing multiple representations in the teaching process on student achievement (Gürbüz & Şahin, 2005). Among these categories, qualitative research designs are commonly employed to study the use of multiple representations (Türer & Günhan, 2022). However, conducting field surveys with quantitative designs, which allow for larger sample sizes, can provide a more comprehensive overview of the current situation. It is worth noting that studies on multiple representations predominantly focus on the field of mathematics (Cleaves, 2008; Çelik, Sağlam Arslan, 2012; Eroğlu & Akkuş, 2021; İncikabı & Biber, 2018; Türer & Günhan, 2022). In addition to mathematics, multiple forms of representation are utilized in various fields, such as physics, chemistry, and geography (Kohl, Rosengrand, & Finkelstein, 2007). However, the emphasis on multiple representations in these disciplines has not been as significant as in mathematics. In physics problem-solving, diverse representations such as pictures, graphs, and diagrams are commonly employed to comprehend the question (Van Heuvelen & Zou, 2001), highlighting the foundational role of multiple representations in physics learning (Kohl, Rosengrand, & Finkelstein, 2007). Notably, secondary school students are predominantly selected as the sample population in studies focusing on multiple representations (Türer & Günhan, 2022). Additionally, there has been an increasing application of multiple representations in teacher education programs (Ryken, 2009; Gürbüz ve Şahin, 2005; Dreher & Kuntze, 2015; Türer & Günhan, 2022). However, a shortage of studies focusing specifically on high school students indicates the need for further development in this area. In line with this, the primary objective of this study is to investigate the multiple representation translation skills of high school students across different grade levels in the context of one-dimensional motion. The sub-problems to be addressed are as follows:

1. How do high school students' transitions from figures, tables, graphs, verbal and algebraic representations to other representations vary?

2. How do high school students' multiple representation transfer skill levels change according to grades?

## 2. METHOD

To assess the progression of multiple representation skills among high school students, involving students from different grade levels is essential. Consequently, the research is aptly suited for a cross-sectional research model. Cross-sectional studies involve observing groups with equal developmental levels in different years instead of following the same sample over a long period, which saves time (Johnson & Christensen, 2019). The sample selection for cross-sectional studies can be cross-age or cross-grade (Lin, 2017), with the former focusing on cognitive development and the latter considering changes in knowledge influenced by the environment and curriculum based on grade levels. In this study, a cross-grade design was employed.

### 2.1 Participants

The participants in this study consisted of 239 students enrolled in the 9th, 10th, and 11th grades. The distribution of these students across grades was as follows: 84 in 9th grade, 106 in 10th grade, and 49 in 11th grade. The study sampled students from the initial three years (9th, 10th, and 11th grades) of the four-year high school education. The exclusion of 12th-grade students was due to their intensive preparation for a nationwide university entrance exam, resulting in limited school engagement and accessibility, making it challenging to reach and involve them in the study. The study was conducted in a small province with a relatively small population in the Black Sea region of Turkey.

In Turkey, there are three types of high schools. The first type includes science, social science, and project high schools, which offer education in science, social science, and a combination of science and social science (the latter course is called the equal-weighted program) and admit students through an entrance examination. The second type consists of Anatolian high schools, which provide education in similar fields but admit students without an examination. In both types of high schools, students choose a specific field at the end of 9th grade. The last type of high school is a vocational high school, which focuses on vocational education. In vocational high schools, physics is taught only in the 9th grade. The study participants were students from an Anatolian high school with general student admission. Since physics, chemistry, and biology courses continued in the 10th grade, the applications and assessments were conducted only with students in the science field.

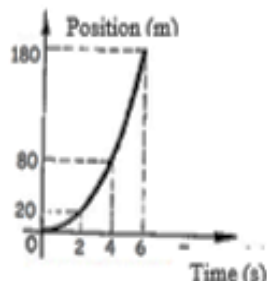
### 2.2 Data collection tool

During the preparation of the data collection tool, an analysis of relevant literature and textbooks was conducted. The focus was on the unit of one-dimensional motion, and the researcher developed a multiple representation translations test consisting of 20 questions. The test used figure, table, graph, verbal, and algebraic representations, respectively.

**Table 1** Content and subject distributions of the questions

Explanation	Translation questions	Subject
Figure	Graphic, Table, Verbal, and Algebraic	Constant speed motion
Graphic	Figure, Table, Verbal, and Algebraic	Vertical motion
Table	Figure, Graphic, Verbal, Algebraic	Free fall
Verbal	Figure, Graphic, Table, Algebraic	Motion with constant acceleration
Algebraic	Figure, Graphic, Table, Verbal	Free fall

**Q3.** Below is the position-time graph of an object in free fall.



**Q5:** The equation describing the motion of an object over time is  $x=5t^2$ , where  $x$  represents the motion and  $t$  represents time. (5 is a constant in  $m/s^2$ )

a) Create a table containing the position and time variables of this object until the moment shown in the graph.

b) Formulate a general expression for calculating the time taken by the moving object to cover a distance and write the name of each variable you use.

c) Draw a figure describing the motion of the object.

d) Express the above graphical data verbally.

a) Draw the path-time graph of this object for the first 4 seconds

b) Create a table containing the position and time of the object for the first 4 seconds

c) Draw a figure showing the motion for the first 4 seconds

d) Express the mathematical situation in the question verbally

**Figure 1** Two examples of a question assessing translating skill from graphic and mathematical representations to other representations

Then, questions were prepared to transform the information in the explanation into the other four representations. The questions prepared by the researcher were examined for face and construct validity by a panel of five physics teachers and one physics teacher and then evaluated by the physics teacher working in the high school where the application would be carried out. Accordingly, some questions in the test were simplified in terms of content, the number values of some were changed, and a new page layout was preferred. The test was finalized by completing the suggested corrections. The subject and content distributions of the questions in the prepared test are given in Table 1.

The questionnaire utilized in this study began by collecting demographic information such as participants' gender and grade level. Subsequently, participants were provided with instructions outlining the purpose of the questionnaire, the content of the test, and guidance on how to respond to the test items. The questionnaire presented a figure, graph, table, verbal explanation, or algebraic expression, followed by subsequent questions. Four questions were presented for each explanation to prompt the transformation of the given information into other representation types. Specifically, in the case of transforming from graphical representation to other forms, Figures 1 display the explanations and translation questions

utilized. Ample space was provided beneath each question to allow participants to write their responses. The questionnaire spanned four pages and was administered to the students.

### 2.3. Data collection process

The data collection process for this study took one week during the spring term of the 2022-2023 academic year. The questionnaire was administered during physics lessons, during which the physics teacher and the researcher were present throughout the data collection period. Before administering the questionnaire in each class, the researcher provided information to the participants regarding the study's ethical considerations. Participants were informed that their involvement in the study was voluntary and had the right to decline to answer any question. It was emphasized that no information exchange or communication among students was permitted during the administration.

Furthermore, students were instructed to complete the questionnaire independently. The questionnaire design allowed for completion within 40 minutes, corresponding to one lesson duration. Even if there was an early finisher, all answer sheets were collected at the end of the lesson.

### 2.4 Data Analysis

In the data analysis of the study, student responses were assessed using an evaluation scale consisting of four

categories, which had been previously utilized by other researchers (Başkan, 2011; Birgin, 2012). This evaluation scale consisted of the following categories: correct, partially correct, incorrect, and blank. A correct answer (3 points) was defined as an explanation encompassing all aspects of the correct answer. A partially correct answer (2 points) included either a portion of the correct answer or an aspect of the correct answer alongside incorrect statements. An incorrect answer (1 point) referred to explanations that were irrelevant and did not contain the correct answer. Blank answers (0 points) were assigned to participants who left the answer blank or did not understand the question or know the answer. The researcher analyzed the data based on these criteria, and subsequently, a second researcher independently re-evaluated and analyzed the responses. To ensure reliability, 48 students, which accounted for 20% of the participants, were randomly selected and transferred to Microsoft Excel. The second researcher conducted a re-evaluation of these students' responses. The inter-rater agreement between the two researchers was calculated using the formula  $[\text{Agreement} / (\text{Agreement} + \text{Disagreement}) \times 100]$ , as suggested by Miles and Huberman (1994). The result of this formula indicated an inter-rater agreement of 88.2%. Since a 70% or higher threshold was defined as a perfect agreement, the coding was deemed reliable based on the obtained agreement percentage. The correlation coefficient analyzes the relationship between two variables; therefore, the agreement between the two researchers was analyzed.

All student results were transferred to the SPSS statistical software package in the data analysis phase. Mean and standard deviation values were calculated separately for each representation situation by class and overall. Additionally, the scores obtained for each translation were analyzed, and the frequencies and percentages of scores within these representations were computed. Finally, the data were analyzed based on class levels using the SPSS package. The total scores achieved by the students were evaluated, and the significance of the scores obtained in

each class compared to the other classes was assessed. To determine if there was a difference between the three different grade level classes, the statistical test "one-way analysis of variance ANOVA" was employed. Here, grades are the independent variables, and multiple representation transfer skills are the dependent variables. The LSD test was used to evaluate whether the groups were significantly different from each other.

### 3. RESULT AND DISCUSSION

#### 3.1. Students' multiple representation transfer skills

The mean scores and standard deviation values for translating figures, tables, graphs, verbal, and algebraic representations into other representations are given in Table 2. The scores obtained by the students from each transfer skill are presented in detail in the appendix.

The mean values based on the grade levels in Table 2 show that the highest mean score in all grades is observed for translation from the figure representation. Specifically, 11th-grade students achieved the highest average score in this category ( $\bar{x}=8.428$ ). Regarding the overall grade averages, the lowest mean score for translation is found in verbal representation for both 10th and 11th grades. For 9th grade, the lowest mean score was obtained in translation from algebraic representation ( $\bar{x}=2.702$ ). While the mean values generally tend to be higher in 11th grade, 10th-grade students obtained the highest mean scores in translation from graphical and algebraic representation ( $\bar{x}=5.490$ ,  $\bar{x}=3.358$ ). When considering all grade levels, the mean scores vary as follows: translation from figure representation ( $\bar{x}=7.531$ ), translation from table representation ( $\bar{x}=5.456$ ), translation from graphical representation ( $\bar{x}=5.125$ ), translation from verbal representation ( $\bar{x}=2.594$ ), and translation from algebraic representation ( $\bar{x}=3.121$ ). Accordingly, the ranking of students' success in translating representations in descending order is as follows: translation from figure, table, graph, algebraic, and verbal representations to other representations.

**Table 2** Descriptive statistics for the translation skills of the students

		From figure representation to others	From table representation to others	From graphic representation to others	From verbal representation to others	From algebraic representation to others
9 <sup>th</sup> grade	$\bar{x}$	7.321	5.059	4.678	2.714	2.702
	S	2.151	1.751	2.415	2.397	3.176
10 <sup>th</sup> grade	$\bar{x}$	7.208	5.330	5.490	2.320	3.358
	S	2.433	2.069	2.889	2.299	4.269
11 <sup>th</sup> grade	$\bar{x}$	8.428	6.408	5.102	2.979	3.326
	S	2.397	2.518	2.793	2.453	3.777
All grades	$\bar{x}$	7.531	5.456	5.125	2.594	3.121
	S	2.365	2.133	2.724	2.371	3.812

N=239

Table 3 displays the frequency and percentage values of the scores obtained by the students. The table provides insights into the scores obtained by students in various translations between different representation types. Notably, students did not receive zero points solely in the translation from figure representation; rather, some students received zero points in all other representation types as well. The highest score in translation from verbal representation to other forms was 8, the highest score achieved in the translation from tables and graphical representations to other forms was 10, and the highest score obtained in the translation from figure and algebraic representations was 12. When each category was examined individually, the highest frequencies were found to be 8 and 9 points for translation from figure representation, 5 points for translation from table representation, 3 points for translation from graphical representation, 1 point for translation from verbal representation, and 0 points for translation from algebraic representation.

**3.2. Multiple representations by grades**

To demonstrate whether students' multiple representation achievement scores varied according to their grade levels, a one-way analysis of variance (ANOVA) was conducted. The ANOVA results are presented in Table 4.

Table 4 indicates a significant difference between students' grades and their multiple representation achievements ( $F=3.969$ ;  $p<0.05$ ). According to the LSD test results for the distribution of students' multiple representation levels across grades, the multiple

representation test scores of 9th-grade students ( $\bar{x}=4.495$ ) were lower than those of 11th-grade students ( $\bar{x}=5.249$ ). However, no significant difference was observed between the other grades.

**Discussion**

This study focused primarily on investigating high school students' multiple representation translation skills and examining how these skills vary across grade levels. The sub-problems within this study were thoroughly examined, leading to insightful discussions and meaningful conclusions.

Addressing the first sub-problem, which explored multiple representation translation skills, the findings indicated that translation from figure representation to other forms exhibited the highest success. This was followed by a translation from tables and graphical representations. In contrast, the least successful type of translation was observed in verbal representation. Previous research on translation skills can be categorized into two main areas: studies conducted with students and studies analyzing textbooks. When considering students' translation skills, variations were observed based on their grade levels. For example, a study conducted with eighth-grade students found that they encountered the greatest difficulty in translating from verbal representation to other forms. In contrast, their most successful translation occurred from other representations to table representations (Sert, 2007). Another study involving 8th-grade students revealed that the most challenging

**Table 3** Distribution of the translation skill scores

Score	Translation from figure representation		Translation from table representation		Translation from graphic representation		Translation from verbal representation		Translation from algebraic representation	
	f	%	f	%	f	%	f	%	f	%
0	-	-	2	0.8	14	5.9	66	27.6	95	39.7
1	3	1.3	4	1.7	12	5.0	41	17.2	27	11.3
2	3	1.3	16	6.7	16	6.7	19	7.9	12	5.0
3	6	2.5	22	9.2	35	14.6	31	13.0	18	7.5
4	16	6.7	26	10.9	24	10.0	19	7.9	25	10.5
5	21	8.8	60	25.1	23	9.6	25	10.5	10	4.2
6	23	9.6	34	14.2	32	13.4	22	9.2	8	3.3
7	36	5.1	37	15.5	30	12.6	13	5.4	7	2.9
8	42	17.6	17	7.1	23	9.6	3	1.3	3	1.3
9	42	17.6	11	4.6	21	8.8	-	-	7	2.9
10	30	12.6	10	4.2	9	3.8	-	-	7	2.9
11	4	1.7	-	-	-	-	-	-	3	1.3
12	13	5.4	-	-	-	-	-	-	17	7.1

**Table 4** ANOVA results of multiple representation achievement scores according to grade level

Grade	N	$\bar{x}$	S	Source of variance	Sum of sq	Std dev.	Mean square	F	p	Significance
9 <sup>th</sup> grade	84	4.495	1.391	Intergroup	17.598	2	8.799	3.963	0.02	9-11;11-9
10 <sup>th</sup> grade	106	4.757	1.519		Intragroup					
11 <sup>th</sup> grade	49	5.249	1.586	Total	541.519	238				
<b>Total</b>	239	4.766	1.508							

translation was from other representations to graphs, whereas the most successful translation was from other representations to table representations (Gürbüz & Şahin, 2015). Additional insights into students' multiple representation skills can be gained from various studies conducted at the high school level. These studies shed light on students' specific challenges and proficiency levels when working with different representations. For instance, in a study focusing on 9th-grade students' multiple representation skills in expressing algebraic concepts of functions, it was found that students demonstrated proficiency in algebraic representations but encountered difficulties with verbal and graphical representations (Baştürk, 2010). Similarly, Mercan (2020) investigated multiple representations among 9th-grade students in the context of equations and inequalities and revealed that students successfully translated representations, with graphical representations being the most proficient, followed by table, verbal, and algebraic representations. In different studies, Lusiyana (2019) and Pebriana, Supahar, Pradana & Mundilarto, (2022) stated that high school students had the most difficulty converting mathematical expressions into representations. At the university level, a study involving prospective pre-service primary school teachers found that these students faced challenges in creating graphical representations and lacked proficiency in utilizing scientific methods, often resorting to repeating answers without proper explanations (Çelik & Sağlam-Arslan, 2012). Another study focused on graph drawing for motion among university students and identified difficulties in creating graphs related to objects and shapes in motion (Kusairi, Noviandari, & Pratiwi, 2019). In the context of the kinematics of motion, a study tasked students with solving a given problem using different representations. The results indicated that students initially attempted to solve the problem algebraically but, when unsuccessful, turned to graphical solutions and verbal representations with limited success (Puspitaningtyas, Hasanah, Kusairi & Purwaningsih, 2021). Additionally, university students encountered challenges when solving verbal questions related to one-dimensional motion, requiring more time and leaving more questions unanswered (Güzel & Yener, 2010). Overall, when examining studies conducted on students, it becomes evident that the most challenging areas lie in the translation from verbal and graphical representations to other forms, which is partly consistent with the current study's findings. Furthermore, the investigation of textbooks concerning multiple representations revealed that algebraic representations were predominantly utilized in secondary school mathematics textbooks (İncikabı, 2017; İncikabı & Biber, 2018; Karakuzu, 2017), and verbal representations ranked second in frequency, while table and graphic representations were employed the least (İncikabı, 2017; İncikabı & Biber, 2018). Consistent with the present study,

a similar pattern emerged when examining mathematics subjects covered in high school textbooks. Figurative representation emerged as the most prevalent representation type, whereas verbal and algebraic representations were the least commonly used (Eroğlu & Akkuş, 2021). However, the 9th-grade mathematics textbooks examined by Baştürk (2007) and the 8th-grade mathematics textbooks examined by Alkhateeb (2019) revealed that algebraic representation was dominant and other types of representation were not sufficiently addressed. Consequently, one possible explanation for students' struggles with verbal representations in this study could be their limited exposure to such applications (Lusiyana, 2019). The insufficient proficiency displayed across representation types indicates an underdeveloped conceptual understanding among students (Baştürk, 2010). The success of figure, table, and graphic representations, observed in alignment with other studies, may be attributed to their incorporation into diverse courses such as mathematics, chemistry, biology, and geography, thereby increasing students' familiarity and usage (Gürbüz & Şahin, 2015). Another factor contributing to the difficulty encountered with different representation types could be the tendency of students to rely solely on a single representation, leading them to solve problems exclusively within that framework and ultimately failing to learn alternative representation types (Saputra, Jumadi, Paramitha & Sarah, 2019).

Significant differences in representation skills were observed between the 9th and 11th grades, with 11th graders demonstrating superior proficiency. However, no significant differences were found among other grade levels. The lower achievement of students in lower grades may be attributed to their incomplete conceptual understanding of physics, particularly in one-dimensional motion (Umrotul, Jewaru, Kusairi & Pramono, 2022). Difficulties with graph interpretation in kinematics, inadequate comprehension of subject content, and insufficient knowledge of graph construction may contribute to lower scores (Foster, 2004). A study investigating kinematics graphs among 9th graders identified a lack of understanding of relevant topics and failure to recognize relationships between graphs as reasons for students' inability to draw accurate graphs (Demirci & Uyanık, 2009). In this study, one possible reason for improving multiple representation skills among higher grade levels may be the reinforcement and consolidation of subject matter as students progress through subsequent grades.

Furthermore, it was observed that students did not exhibit significant progress in consecutive grade levels, except for a significant development in inter-representational skills between the 9th and 11th grades. Students' limited improvement in inter-representational translation skills may be attributed to perceiving these skills

as difficult, unnecessary tasks or time-consuming activities (Van Heuvelen & Zou, 2001). Another contributing factor to the observed failures may be the prevalence of multiple-choice question formats in general exams, such as the university entrance exam in Turkey. The educational system's emphasis on exam-oriented structures throughout the learning and training process could also be a significant aspect of this issue (Gürbüz & Şahin, 2015), which leads to an exam-oriented educational approach. This examination format often neglects the assessment of higher cognitive domains, thereby hindering the development of students' conceptual understanding.

#### 4. CONCLUSION

The key finding of this study highlights the lack of success among high school students in effectively utilizing multiple representations to express the concept of one-dimensional motion, indicating insufficient knowledge and experience in this area. The ability to translate between representations is influenced by the types of representations employed in classroom instruction and the level of familiarity students have with them. Furthermore, the success rates demonstrated an upward trend with increasing grade levels, suggesting a positive correlation between subject learning and multiple representation translation skills development. This finding underscores the interdependence between subject comprehension and the ability to translate knowledge across different representation types effectively.

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## Appendix

	Evaluation scale	0			1			2			3		
		Grade	9 <sup>th</sup>	10 <sup>th</sup>	11 <sup>th</sup>	9 <sup>th</sup>	10 <sup>th</sup>	11 <sup>th</sup>	9 <sup>th</sup>	10 <sup>th</sup>	11 <sup>th</sup>	9 <sup>th</sup>	10 <sup>th</sup>
Translation from figure	Verbal	2	5	1	32	46	17	6	7	3	44	48	28
	Graphic	0	2	1	20	27	9	2	4	3	62	73	36
	Algebraic	49	33	14	26	58	13	2	6	4	7	9	18
	Table	12	11	1	21	30	16	2	5	4	49	60	28
Translation from table	Graphic	1	4	2	22	21	7	21	30	6	40	51	34
	Verbal	3	10	5	39	40	12	11	8	1	31	48	31
	Figure	20	21	6	58	70	26	2	9	5	4	6	12
	Algebraic	72	29	35	12	76	14	0	1	0	0	0	0
Translation from graphic	Table	14	19	9	24	19	7	3	7	2	43	61	31
	Algebraic	66	65	24	18	41	25	0	0	0	0	0	0
	Figure	28	21	8	32	34	20	6	32	11	18	19	10
	Verbal	23	24	19	28	25	7	6	16	12	27	41	11
Translation from verbal	Graphic	38	41	18	30	40	19	7	14	8	9	11	4
	Table	38	61	30	41	41	18	0	1	1	0	3	0
	Figure	43	66	24	15	16	5	0	6	2	26	18	18
	Algebraic	71	89	39	13	17	10	0	0	0	0	0	0
Translation from algebraic	Graphic	46	48	21	24	27	12	3	1	2	11	30	14
	Table	53	73	27	23	9	14	1	0	0	7	24	8
	Figure	59	72	32	15	14	9	2	2	2	8	18	6
	Verbal	48	66	34	13	15	5	6	5	2	17	20	8