

# ANALYSIS OF STUDENTS' EPISTEMOLOGICAL OBSTACLES ON THE SUBJECT OF PYTHAGOREAN THEOREM

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## ABSTRACT

This article explains mistakes made by junior high school students in solving Pythagorean Theorem problems as well as analysis of students' difficulties in applying mathematical concepts (epistemological obstacles), in which these were reflected in their test answers. This research used descriptive exploratory method to describe symptom and phenomena, which occurs within the students' mind while solving the problem. Data were obtained from a test of Pythagorean Theorem problems given to 99 students from three different schools clusters. Results suggested that students tends to use a quick way to solve the problems without adequate understanding of the concept, remember the term alone without a profound understanding of the concept, fail in solving the problems with implicit information and the problem that requires visual representation in the process of solving the problem. Students also did not like the word problems or a problem with long questions. Teachers play a pivotal role in helping the students to overcome epistemological obstacles such as by giving the students more exercises as well as using different tools and technique in teaching the concept.

**Keywords:** learning obstacles, epistemological obstacles, pythagorean theorem

## ABSTRAK

Artikel ini menyajikan kesalahan yang dilakukan oleh siswa sekolah menengah pertama dalam menyelesaikan soal-soal terkait teorema pythagoras dan juga analisis mengenai kesulitan siswa dalam mengaplikasikan konsep matematik (*epistemological obstacle*). Kesalahan-kesalahan ini dapat dilihat dari jawaban yang diberikan siswa pada tes yang diberikan. Penelitian ini menggunakan pendekatan deskriptif untuk menjelaskan fenomena yang terjadi pada siswa dalam menyelesaikan soal-soal terkait teorema Phytagoras. Data yang diperoleh pada penelitian ini berasal dari hasil tes soal-soal Teorema phytagoras yang diberikan pada 99 siswa dari tiga *cluster* sekolah yang berbeda. Hasil analisis menunjukkan bahwa siswa cenderung menggunakan cara cepat untuk menyelesaikan persoalan tanpa memahami konsep, mengingat rumus secara utuh tanpa pemahaman yang mendalam terkait konsep phytagoras, gagal menyelesaikan soal yang memuat informasi yang implisit dan soal yang memuat informasi yang disajikan secara visual. Hasil wawancara menunjukkan bahwa siswa tidak suka soal cerita dan soal yang disajikan secara panjang lebar. Guru memainkan peran yang sangat penting dalam membantu siswa untuk mengatasi *epistemological obstacle* seperti dengan memberikan siswa latihan soal serta menggunakan berbagai media dan teknik pengajaran dalam mengajarkan konsep.

**Kata kunci:** kesulitan belajar, kesulitan epistemologis, teorema phytagoras

## INTRODUCTION

Learning occurred in the process of coding and encoding of information in students' mind in which students will absorb the knowledge in different ways depends on some factors such as physiological, social, emotional, intellectual and pedagogical factors which in turn may cause learning obstacles. Prior knowledge is intellectual factor instances which is greatly

affect the success of student learning. Students with sufficient prior knowledge are likely to accept new information, (relating) new knowledge with previously obtained knowledege, manipulating them in the learning experiences with ease, as well as having more confidence in solving problems. On the contrary, students with less prior knowledge will have difficulties in accepting new information because they tend to probe the concept, due to their lack of

knowledge about basic concept, so that they will face difficulties in understanding the concept. In the learning process, Brousseau (1997) defined three types of learning obstacles, namely ontogenic obstacles, didactical obstacles and epistemological obstacles. Ontogenic obstacles are obstacles due to discrepancy between students' and teachers' level of knowledge. Didactical obstacles are caused by the lack of precise methods or approach that teacher use in teaching, and the epistemological obstacles are obstacles which associated with knowledge and how knowledge is acquired, in which it was caused by the mathematical concept. In describing epistemological obstacles, Brousseau (1997, pp. 87) stated that "Obstacles of really epistemological origin are those from which one neither can nor should escape, because of their formative role in the knowledge being sought".

Epistemology is associated with the knowledge itself and how knowledge is acquired. Spagnolo (in Vankus, 2005) stated that "Epistemological obstacles come from the nature of the concept that has to be taught. For instance, if there are some non-continuity or radical changes in the evolution of mathematical concept, epistemological obstacles during the teaching of this concept could appear". Epistemological can be seen from the way students solve certain problem. Eventhough student may be able to answer problem with a certain concept, it does not necessarily means that the student doesn't have epistemilological obstacles. Therefore, solving problem in different context within the same concept can reveal whether the obstacles do exist.

In Indonesian mathematics curriculum Year 2006 as well as Year 2013 for junior high school, Phytagorean Theorem is one of the pivotal standard competency in which the students are required to be able to use this Theorem in solving mathematical problems (Kemendiknas, 2006; Kemendikbud, 2013). This demand cannot be achieved when students still have difficulties in solving a problem. Therefore, this paper will focus in an in-depth analysis of learning obstacles in students especially describing the epistemological obstacles of junior high school students on the subjects of Phytagorean Theorem.

## METHOD

The method used in this research was descriptive exploratory method. Descriptive method was used because of several reasons, 1) the collected data are not in a form of number, but rather interview results, field notes, and documents, and 2) epistemological obstacles are discussed in a complex and holistic way.

Schools in Indonesia are divided into three school clusters, therefore to ensure that our data is representative, students involved in this study were 38 students from first (high), 29 students from second (moderate), and 32 students from third (low) cluster school. Each student was given a test with six (6) Pythagorean Theorem problems. From total students given the Pythagorean Theorem problems, some were selected for an in-depth interview to gain a deeper understanding of the problems experienced by students in solving the given problem. Those students were selected based on their test answers, i.e. the obstacles they seemed to experience.

## RESULTS AND DISCUSSION

Results suggested that misconception, implicit information, and visual representation were factors influencing epistemological obstacles, in which from six problems given to the students, misconception was apparent in Problem One, implicit information in Problem Four, and visual representation in Problem Six.

### *The Influence of Misconception on Epistemological Obstacles*

In solving Problem One (Figure 1), the first thing that the students have to do was to determine the longest side of the triangle. Secondly, they need to know the meaning behind the Pythagorean Theorem which is that "*the square length of hypotenuse is equal to the sum of the square length of other sides.*" Thirdly, they need to draw a conclusion whether the sides is considered as Pythagorean triple or not.

Problem One

Data below are sides length of triangles. Determine which one is a Pythagorean Triple and state your reason(s)!

3 cm, 4cm, 5cm
15cm, 20cm, 25 cm
6 cm, 8 cm, 10 cm
9 cm, 12 cm, 14 cm

**Figure 1. Problem One**

Results suggested that students started solving the problem from the second step. Judging by the answer, there are two possibilities that can be drawn. First, students indeed understand the uses of  $c^2 = a^2 + b^2$ : they already knew that c is the longest side of a triangle. Second, students used  $c^2 = a^2 + b^2$  because the given problem is about Pythagorean Theorem, eventhough they do not actually know how to use the formula in the problem context. The difference between the first and second possibility is the conclusion, in the first possibility, students will draw a conclusion regarding the longest side of a triangle, but in the second possibility, students will conclude nothing in the end, because they do not actually know what they are working on.

There are five kinds of obstacles found from the students' answers; those can be clearly seen on the following explanation.

*The First Obstacle in Problem One*

To address the first obstacle in Problem One, answers from Student 11 (S11) and Student 37 (S37) will be discussed here. They both start from the second step by wrote Pythagorean Theorem  $b^2 = a^2 + c^2$  (b as the hypotenuse). S11 then substituted the known sides' lengths in the Pythagorean Theorem by choosing the longest side as the 'sisi miring' (the commonly used term). S11 obtained the result that  $5^2$  is equal to  $3^2 + 4^2$ , so that S11 conclude that 3, 4 and 5 are Pythagorean triple, with the explanation that the square length of the hypotenuse is equal to the sum of the square length of the other sides. Likewise for 1b and 1c, S11 conclude that both are Pythagorean triples, while 1d is not a Pythagorean triple because  $14^2 \neq 9^2 + 12^2$ .

S11 used the term 'sisi miring' not the longest side (*sisi terpanjang*), in which the hypotenuse was not known yet in the problems, students only knew that hypotenuse is the longest side of a triangle. So we conducted an interview to S11 regarding her answer and she said:

*"because the formula was like that, mam!, the square of hypotenusa length is equal to sum square of the other two length"*

This suggested that she used the formula because it is the formula of Pythagorean Theorem, so we asked again about how to know which sides is the hypotenuse, and she could not answer. S11 explained that she can decide which one is the hypotenuse by doing a trial and error on her scrap paper. S11 tried every side length as the hypotenuse and substituting it to Pythagorean Theorem and found that  $5^2 = 3^2 + 4^2$ , and this is the reason why she conclude that 3, 4 and 5 are Pythagorean triple. This obstacle derived from the existence of disconnectedness between the Pythagorean Theorem with a right triangle. This shows that in the student mind existing knowledge were not mapped onto the new knowledge. Therefore, comparison of angle-based right triangles should also being given in learning obtuse and acute triangle, so that students understand differences between right triangle and the other two triangle kinds.

The similar obstacle is found on S37's answer that substituted the known sides' lengths in the Pythagorean Theorem and obtained the result that 1a is a Pythagorean triple, because:

*"if 5 cm serves as hypotenuse, 4 cm as vertical length and 3 cm as horizontal length, so when we use Pythagorean Theorem the result will be the same"* (If 5cm serves as the hypotenuse, 4cm is the opposite and 3cm is the adjacent, when we use the Pythagorean formula the result is equal.)

Equal results are also obtained for 1b and 1c, and therefore 1d is not a Pythagorean triple because:

*"the use of Pythagorean Theorem for this problems resulted in different results"* (if the problem use Pythagorean Theorem then the result is not equal).

From the result of the interviews, it was found that S37 attained the answer for 1b by substituting length side one by one as the hypotenuse ( $c^2 = a^2 + b^2$ ), just like S11 did by trial and error. This is reinforced by the fact that S37 wrote 'if 5cm is the hypotenuse' means S37 did not sure whether 5cm is the hypotenuse or not, and that means S37 did not know that to find hypotenuse is to choose the longest side of a triangle.

#### *The Second Obstacle in Problem One*

Student 27 (S27) answers will be discussed here. S27 started from the second step by substituting the sides' lengths in the Pythagorean Theorem  $c^2 = a^2 + b^2$ ; however the use of this formula is not based on the understanding of the Pythagorean Theorem. S27 substituted 3, 4 and 5 in the Pythagorean Theorem, then obtained the answer  $5^2 = 3^2 + 4^2$ , with the reason because  $c^2 = a^2 + b^2$ . Likewise for 1b and 1c which are Pythagorean triple and 1d is not Pythagorean triple because  $c^2 \neq a^2 + b^2$ . S27 did not start by choosing the longest side to be substituted in Pythagorean Theorem, but she substitute each side length. This suggested that she received this concept as a product without going through the process of repersonalization. S27 regarded that every Pythagorean Theorem problems must be solved start from  $c^2 = a^2 + b^2$  formula regardless the problem context. This is reinforced by the interview result in which S27 regarded hypotenuse as the sloping side and agreed to the statement that if there are two sloping sides in right triangle then the triangle has two hypotenuses. This obstacle can be prevented by giving enough identification of right triangle characteristics and also by avoiding giving ambiguous terms to the students because it might cause misconception.

#### *The Third Obstacle in Problem One*

Student 10 (S10) answers will be discussed here. S10 started solving the problem from the third step which is concluding the answer that 1a is a Pythagorean triple while 1b, 1c and 1d are not Pythagorean triples. For 1a, S10 wrote 3, 4, and 5 are Pythagorean triple because they will form a right triangle, then

substituted the known sides' lengths in the  $c^2 = a^2 + b^2$  formula..

In the interview session, S10 was asked further about how she knew that they will form a triangle. Then, we asked again whether it can be drawn to show the formed triangle, but S10 answered that "it cannot be". So, we asked about the hypotenuse of right triangle and S10 answered the same as the previous student that "hypotenuse is a sloping side of a right triangle". This obvious fact suggests that student answered the problem without adequate knowledge and understanding of the Pythagorean Theorem. Besides, S10 answered incorrectly in 1b and 1c whilst the result S10 obtained are  $25^2 = 20^2 + 15^2$  and  $10^2 = 6^2 + 8^2$ . So, it can be concluded that she did not know the link between right triangle and Pythagorean Theorem.

To solve this obstacle, teacher has to give more explanations about right triangle-Pythagorean Theorem relation and give them more tasks to exercise this concept of connectedness. Watson and Mason (2006) stated that teachers could assist students to make connections by using carefully sequenced examples, including examples of students' own solution strategies, to illustrate key mathematical ideas. By progressively introducing modifications that build on students' existing understanding, teachers can emphasize the links between different ideas in mathematics.

#### *The Fourth Obstacle in Problem One*

Student 22 (S22) answers will be discussed here. The next obstacle found was in students' process to communicate the knowledge they already have into words, which makes it hard for them to conclude the answer. Students have not made an investigation that involved reasoning and communication, so that their ideas were not expressed well.

S22 started the process from the second step by substituting the known sides' lengths to  $c^2 = a^2 + b^2$  formula. From the substitution process, she obtained that 1a, 1b, and 1c are Pythagorean triples "because if it were added the result will be same/equal", and 1d is not a Pythagorean triple because "if it was added the result will not be same/equal".

$$3^2 + 4^2 = 9 + 16$$

Figure 2. The Answer of S22

S22 assumed that 25 are equal to  $5^2$  which is the result of  $3^2 + 4^2$  (Figure 2). She was then asked why she chose  $3^2 + 4^2$  instead of  $5^2 + 3^2$  or  $5^2 + 4^2$ . She said that she assumed that the result will not be equal, and this assumption was based on her trial and error on a piece of paper. Just like previous students, S22 did trial and error to find the equal answer, because she only remembered the term that 'the result will be equal' without further information what things are equal in the context. So, when she was asked why it is called Pythagorean triple if she get the equal result, S22 could not answer further.

The same case happened with S53; he started solving the problem by substituting the known sides lengths in Pythagorean Theorem, then wrote the reason as "because the result is same/equal". This was similar with S22, he did not know the concept of Pythagorean Theorem, let alone Pythagorean triple. This is reinforced by the fact that S53 did not know how to determine a hypotenuse from a right triangle, even though he did remember  $c^2 = a^2 + b^2$  formula, he was unmindful with the meaning behind those symbols.

*The Fifth Obstacle in Problem One*

Student 99 (S99) and student 67 (S67) answers will be discussed here. They used the 3, 4 and 5 to identify the sides' lengths. S99 said that they were given some Pythagorean Theorem to be memorized, such as 3, 4, 5 and 6, 8, 10. To determine another Pythagorean Theorem is by finding the multiplication of 3, 4 and 5. Just like 6, 8 and 10, that can be determined by multiplying 3, 4, and 5 with 2. With that knowledge, they determine answer for each question, and found that 1a, 1b, 1c is Pythagorean triple. But their way is not necessarily true, since not all of Pythagorean triple is the multiplication of 3, 4 and 5. For example, a triangle with the sides lengths of 33, 56 and 65 is a Pythagorean triple, but it is not necessarily means that 33, 56, and 65 are determined by multiplying 3 and 4 with 11.

This kind of way can probably help student in answering multiple choice problems that require short amount of time. Like the answer from S67 that assumed all the Pythagorean Theorem is the product of 3, 4 and 5 multiplied by 2. There is a tendency for only remembering a quick way to solve the problem without knowing the reason and the correct procedure of the concept. This lead student to epistemological obstacles where student can only solve the problem with one certain context and when the problem is in a different context from the said one, students could not answer it correctly.

S67 answered Problem One by determining the multiplication of 3, 4 and 5:

"Because Pythagorean triple 3 and 4 is 5, so next r value is  $35:5 = 7$ .

$$AB \text{ Length} = 3 \times 7 = 21,$$

$$BC \text{ Length} = 4 \times 7 = 28"$$

S67 determined the value of r based on the knowledge that Pythagorean triple is the multiple of 3, 4 and 5, however there are Pythagorean triples that are not the multiplication of 3, 4 and 5 such as: 7, 24, 25; 8, 15, 17; 5,12, 13 and other Pythagorean triples. This lead S67 to solve the problem only by using the t of 3, 4 and 5, whilst on this problem students are required to do problem solving using Pythagorean Theorem and algebra. This case also proves that there are ontogenic obstacles within the students.

*The Influence of Implicit Information on Epistemological Obstacles*

Epistemological obstacles may occurs from the implicit information within the problem, so that students often confused by the lack of information that are given. Students obviously manipulate only the explicit information known in the question.

ABCD is a square, its lengths (p+q) are as in the figure below. Determine area of EFGH square

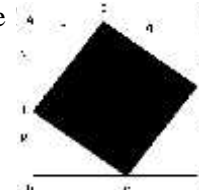


Figure 3. Problem Four

In problem four (Figure 3), the only known value is the length of the ABCD square, which is  $(p + q)$ , this leads the student to two different kinds of problem solving. The first is using Pythagorean Theorem and the other is using the formula of triangle and rectangle area.

To answer the problem using Pythagorean Theorem, students need to know the characteristics of the triangle, such as the equal and equilateral angles. That information is not mentioned in the problem, so students need to describe it first before comes up with the next step. Whereas to answer the problem using the other way, students need to recall the formula of rectangle and triangle area. There are three kinds of obstacles found on the students' answers, those can be clearly seen on the following explanation.

#### *The First Obstacle in Problem Four*

Student 31 (S31) answers will be discussed here. The obstacles occurred is that students could not come up with problem solving mechanism either using Pythagorean Theorem or the triangle/rectangle area formula. In the interview, S31 said that the area of EFGH cannot be determined because there is no known value in the problem. S31 thought that the question is incomplete, so that student could not move on to the next step of solving the problem.

*"How do we find the area if the problem is incomplete, mam?"*

This suggested that S31 did not see the implicit information within the question. Student assumed that the term of area is always about numbers, so when it comes to variables like  $p$  and  $q$ , student cannot work on the problem.

#### **The Second Obstacle in Problem Four**

Student 99 (S99) answers will be discussed here. The lack of geometry knowledge in the students made them answer incorrectly. S99 assumed that to determine the area of EFGH, he need to determine the side length first. Unfortunately, S99 used the wrong formula in determining the side length, because instead of using Pythagorean Theorem, he used the formula of triangle area.

S99 then obtained the area of AEH and assumed that the area of AEH is equal to the EH length. In the end S99 determined the area of EFGH by multiplying the area of AEH with the area of AEH (Figure 4).

Assuming side length is equal to the triangle area is a proof that student could not recognize the elements of a triangle, which then lead to not only an epistemological obstacles but also ontogenic obstacles. Students are required to have the level of relational in Van Hiele level geometry development, but the fact is that they still found difficulties in developing the first level of geometry development.

$$\begin{aligned} \text{L}\Delta A &= \frac{p \cdot q}{2} \\ \text{L} E &= \left(\frac{p \cdot q}{2}\right) \left(\frac{p \cdot q}{2}\right) \\ \text{L} E &= \frac{p^2 + q^2}{4} \end{aligned}$$

**Figure 4. The Answer of S99**

S99 used the formula of triangle area because he did not know that AEH is a right triangle, which means that hypotenuse or the side length of EFGH can actually be determined by using Pythagorean Theorem. S99 could not determine AEH as a right triangle, because he did not know that the angle of ABCD rectangle is  $90^\circ$ . This suggested that students' knowledge about right triangle is not yet adequate and the students' preliminary knowledge is still limited.

#### *The Third Obstacle in Problem Four*

The obstacle that arose in this problem is regarding the students' algebra knowledge. Some students succeed in determining the side length of EFGH by using Pythagorean Theorem, but mostly failed in the process of square roots operation. This can be seen in Student 79 (S79) answer (Figure 5).

$$\begin{aligned} E^2 &= p^2 + q^2 \\ E &= p^2 + q^2 \end{aligned}$$

**Figure 5. Answer of S79**

It will be hard for students to obtain the correct answer for Pythagorean Theorem problems if the students did not master the square roots operation, since all of Pythagorean Theorem processes require this operation. Other incorrect answers in terms of square and roots square operations were that of student 93 (Figure 6) and student 94 (Figure 7).

$$\begin{aligned} \text{L.EFGH} &= \sqrt{p^2 + q^2} \times \sqrt{p^2 + q^2} \\ \text{L.EFGH} &= p + q \times p + q \\ \text{L.EFGH} &= p^2 + q^2 \end{aligned}$$

Figure 6. The Answer of S93

$$\begin{aligned} G^2 &= p^2 + q^2 \\ &= \frac{p^2 + q^2}{G^2} \end{aligned}$$

Figure 7. The Answer of S94

Obstacles above showed that most students did not answer the problem because of their lack of geometry ability and algebra knowledge which both lead to the occurrence of epistemological obstacles and ontogenic obstacles.

**The Influence of Visual Representation on Epistemological Obstacles**

In solving Problem Six (Figure 8), communication ability is required for students to understand what the question is about, what is known and what is being asked. Some of the students failed in understanding the problem, and they inaccurately point out the thing that is told in the question. The obstacle often arises in doing visual representation, which is derived from the lack of knowledge about geometry, such as line and angle. This problem can be solved by making the visual representation of the said question correctly, in which after that, students need to manipulate the comparison of special angle-based right triangles.

**Problem 6**

There are two spotlights A and B. They are set with different height on the same pole which perpendicular over land. The spotlights were thrown to the same point on the land. The spotlight A ray creates 45° angle with the land, whereas spotlight B ray creates 30° with the land. If spotlight B has 4m height over land, what is the height difference between spotlight A and B?

Figure 8. Problem 6

There are three kinds of obstacles found on the students' answers, as will be discussed in the following explanation.

*The First Obstacle in Problem Six*

Student number 92 (S92) drew the illustration of the two towers in the question, but S92 failed in manipulating the comparison of special angle-based right triangles. Interview result suggested that S92 making mistake in using the comparison, because she only memorized the comparison without the knowledge of how it was found, so she tend to follow the comparison she memorized which has different shape of the right triangle. Since students tend to follow the one they memorized, they did not pay attention to the position and just used the comparison that leads them to wrong answers.

The comparison of 45° angle-based right triangle is  $x : x : x\sqrt{2}$  where  $x\sqrt{2}$  is hypotenuse, while the comparison of 30°/60° angle-based right triangles is presented in Figure 9.

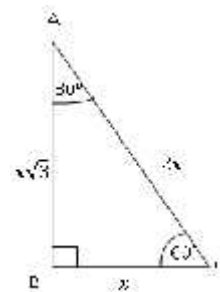


Figure 9. The Comparison of 30°/60° Angle-based Right Triangle

S92 did a lot of mistakes because they tend to memorize the comparison alone without understanding the meaning of the comparison. S92 memorized  $x:2x:x\sqrt{x}$  formula but did not know the context of the comparison, just like how S92 put the  $2x$  on the side beside the  $30^\circ$  angle that actually should be put in the hypotenuse (Figure 10).

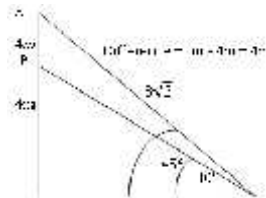


Figure 10. The Answer of S92

*The Second Obstacle in Problem Six*

Student number 62 (S62) tried to solve the problem via an analogy. Because in the question it is known that spotlight B formed a  $30^\circ$  angle and it was attached to the tower 4 meters from the ground, S62 concluded that every  $15^\circ$  angle will be attached to the tower 2 meters from the ground, and because spotlight A formed  $45^\circ$  angle to the ground, then the height of spotlight A is 6m (Figure 11).

$$\begin{aligned}
 4m : 2 &= 2m \\
 30^\circ : 2 &= 15^\circ \\
 &= \frac{A}{1} \times 2m = \frac{4}{1} \times 2m = 6m
 \end{aligned}$$

Figure 11. The Answer of S62

The similar case was found in S66's answer (Figure 12).

$$\begin{aligned}
 \frac{4}{x} &\times \frac{30}{45} \\
 x &= 6m
 \end{aligned}$$

Figure 12. The Answer of S66

Both answers were wrong and showed that students did not manipulate Pythagorean Theorem to solve the problem and they also did not try to make a visual representation that is needed.

*The Third Obstacle in Problem Six*

Some students followed the right solution by making visual representation and then manipulating the comparison of special angle-based right triangles so that they can measured height difference of the spotlights. Unfortunately, students made mistake in making illustration for the question.

Students were less thorough in reading the question especially for "both spotlight were thrown to the same point on the land" statement, so that they assumed that the spotlight were emitted to the different direction (Figure 13).

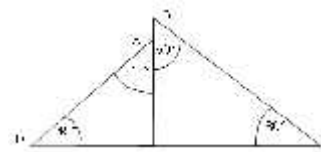


Figure 13. The Answer of S79

Student number 77 (S77) drew the illustration with wrong angle placement, she did not read the question carefully especially for "spotlight A ray creates  $45^\circ$  angle with the land" statement, so the student drew wrong illustration (Figure 13).

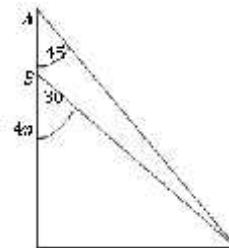


Figure 13. The Answer of S77

Another example is from S86 answer (Figure 14). She drew the illustration of the right triangle that has  $45^\circ$  and  $30^\circ$  angles, which is impossible. This showed that she did not understand that the angle of a triangle is  $180^\circ$ .

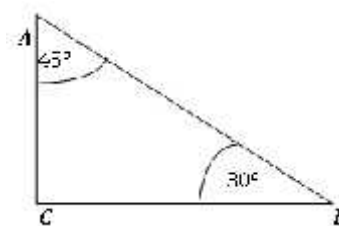


Figure 14. The Answer of S86



Another kind of obstacle is derived from the lack of geometry knowledge like the definition of line and angle. Without adequate geometry knowledge, it will be difficult to draw and make illustration of the question; this was found in some students, among them is S74 (Figure 15).

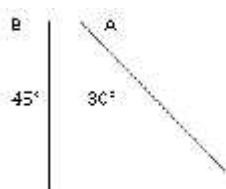


Figure 15. The Answer of S74

Some students even did not answer Problem Six because they confessed that they do not like word problems because it often confused them and tends to have complicated answers.

*“no mam, it will be better if the questions is in a form of calculation. You know, questions that already has clear information about what is known and what is being questioned, so that we don't have to stretch our head just to read the question “*

In the interview, students were asked to read the problem very carefully and asked to draw the illustration of the said question, some students did well in the visual representation but they still have difficulties in solving the problem regarding the comparison of special angle-based right triangles. Some students said that they never learned this before, so they did not know how to find the hypotenuse when one of the side and the length are known. Difficulties in making representation can be overcome if teachers use a range of representation and tools, in which Anthony and Walshaw (2009) stated that using a range of representation and tools can support learners' mathematical development. Furthermore, modeling activities could foster the students in make sense of both contexts and the mathematics embedded in the tasks (English, 2006; Galbraith, Stilman, Brown, & Edwards, 2007).

Epistemological obstacles may occurred due to implicit information within the problem, so that students often confused by the lack of information that are given. Students obviously manipulate only the explicit information known in the question. To overcome this obstacles teacher

have to give the student more problem with implicit information, so the students will become more accustomed in solving this kind of problems. All in all, to overcome learning obstacles especially epistemological obstacle teacher has central responsibility, because no matter how good their teaching intentions is, teacher themselves has to work out how they can best help their students to grasp core mathematical ideas (Hill et al., 2005).

## CONCLUSION

Mistakes in solving Pythagorean Theorem related-questions and difficulties in applying mathematics concepts (epistemological obstacle) stemmed from students' inadequate understanding of the concept in which it made them unable to solving problems in different contexts. Their inability to extract implicit information and making visual representation as well their tendency to avoid word problems or a problem with long questions further hindered their ability to solve the given problems.

Teachers play a pivotal role in helping the students to overcome epistemological obstacles such as by giving the students more exercises as well as using different tools and technique in teaching the concept.

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