

DEVELOPING MATHEMATICAL COMMUNICATION AND REPRESENTATION OF STUDENTS GRADE VII: A DESIGN RESEARCH

Dadang Juandi dan Al Jupri

Department of Mathematics Education,
Faculty of Mathematics and Science Education,
Universitas Pendidikan Indonesia

ABSTRAK

Penelitian ini berfokus pada komunikasi matematika dan representasi melalui pembelajaran berbasis masalah yang dikembangkan melalui penelitian desain. Ada dua tujuan dari penelitian ini. Pertama, dalam jangka pendek, penelitian ini dilakukan untuk mengetahui: bagaimana komunikasi matematika dan representasi siswa berkembang, apakah budaya kelas mendukung pencapaian kompetensi dasar-, dan apa hambatan siswa untuk belajar. Kedua, dalam jangka panjang, penelitian ini dilakukan untuk mengembangkan “teori” dalam pendidikan matematika. Kedua tujuan tersebut diharapkan dapat dicapai melalui penelitian pengembangan (dalam hal ini menggunakan desain penelitian). Tujuan pertama dicapai dengan desain penelitian yang meliputi tiga tahap: desain awal, eksperimen, dan analisis retrospektif. Penelitian awal menunjukkan bahwa: (i) komunikasi matematika dan representasi dapat dikembangkan melalui bahan pembelajaran kontekstual, intervensi guru yang tepat itu, berbagai pengaturan belajar-mengajar, situasi didactical dikembangkan oleh guru, dan usaha guru dalam menghubungkan situasi didactical yakni antara bahan dan siswa belajar, dan antara mahasiswa dan guru, (ii) kesulitan siswa meliputi: kesulitan dalam berkomunikasi ide matematika secara lisan, mewakili ide matematika aljabar, menggunakan representasi matematis untuk memecahkan masalah, dan mengusulkan argumentasi, dan (iii) kesulitan guru adalah menganalisis komunikasi matematika dan representasi siswa.

Kata Kunci: hambatan belajar komunikasi matematika, rancangan penelitian, representasi matematika

ABSTRACT

This research studies mathematical communication and representation through problem based-learning which is developed by design research. There are two objectives of this research. First, in short term, the research is conducted to know: how students' mathematical communication and representation develops, does classroom culture support the achievement of basic-competence, and what are students' learning obstacles. Second, in long term, this research is conducted to develop “a theory” in mathematics education. These two objectives are expected to be achieved through developmental research (in this case using design research). The first objective is achieved by design research which includes three phases: preliminary design, experiment, and retrospective analysis. The preliminary research showed that: (i) mathematical communication and representation could be developed through contextual learning materials, proper teacher's intervention, various learning-teaching settings, a didactical situation developed by the teacher, and the teacher's effort in connecting didactical situations i.e., between learning materials and students, and between students and the teacher; (ii) students' difficulties include: a difficulty in communicating mathematical idea verbally, representing mathematical idea algebraically, using mathematical representation to solve problems, and proposing argumentation; and (iii) the teacher's difficulty is analyzing students' mathematical communication and representations.

Key Words: *learning obstacles, mathematical communication, mathematical representation, design research*

INTRODUCTION

It has been widely known that students will learn mathematics actively if it happens in learning environment that support

negotiation to achieve meaningfulness. And, whatever conditions, teacher's role is very important to achieve that meaningfulness. One of the teacher's roles is to develop mathematical communication in his/her class.

Mathematical communication will happen properly if the teacher prepares it. According to Emori (2005), a good learning process is developed based on students' natural thinking. According to Suryadi (2006), to know students' thinking process, the teacher should know how to construct problem, instruct learning line, and using learning media as a representation tools.

Posing questions both by students and teacher in learning-teaching process is an activity that should always exist—where this means that students actively participate in knowledge formation. The questions that posed should be directed to achieve learning objectives. Sabandar (2005) states that, “If students are expected to think critically and creatively in learning-teaching mathematics process, then posing challenging question—which is a divergent question, or making cognitive-conflicts should be raised.” Therefore, one possibility to elicit students' communication ability is by posing various challenging questions—where the questions should be in line with students development.

Observation in several Junior High Schools in three different regencies in West-Java on the implementation of Indonesian Kurikulum (KTSP) showed that the learning-teaching processes are still teacher-centered, the teachers did not use learning media—so that students had less experiences in manipulating objects that represent mathematical objects (Hendayana, 2007). In other words, mathematical communication and representation in mathematics learning-teaching processes is not optimally exploited to enhance students' understanding, therefore, it should be developed.

Mathematical communication and representation are abilities that should be achieved by students according to Indonesian curriculum (depdiknas, 2006). Mathematical communication is a way to clarify understanding and share ideas in learning-teaching mathematics. In learning mathematics, ideas that emerge from problem solving processes can be used as reflection, discussion, and change (NCTM, 2000). Students will have a chance to try thinking when they are given a challenge. Various

ideas and obstacles that emerge during learning-teaching process are potential sources for discussion, comparison, or clarification. In discussion there will be interaction between students, this can be a potential to develop mathematical communication and representation of students. The importance of these abilities is also supported by Linquist (1996), Esty&Montana (1996), Greenes & Schulman (1996), Usiskin (1996), and Riedesel, Schwartz & Clements (1995).

Mathematical representation is useful either for students or for the teacher. For students, representation can be used for: understanding mathematical concepts and its relations; communicating mathematical approach, shares ideas to students themselves and other people; recognizing connection between concepts; applying mathematics to realistic problems, helping to solve problems, and giving opportunities to represent mathematical ideas meaningfully. For teachers, representation is used to obtain valuable insight from students' works and bridge students' representation to conventional way if necessary (NCTM, 2000). In order to obtain opportunity to think, communicate, discuss, and represent students ideas, teachers should facilitate them with mathematics problems—either procedural problems or non-routine problems.

One learning-teaching approach that potentially gives many opportunities to activate students in discussion, communication, and facilitate the development of mathematical representation is problem-based learning (PBL). PBL is the learning approach that based on constructivism—which assumes that understanding will emerge through interaction with learning environment, and cognitive conflicts (a stimulus for learning that determines what should be learned) (Savery and duffy, 1996).

Since mathematical communication and representation are important, then it is also important to encourage students to communicate and represent mathematical ideas through the ways that they had understood—even if their ways are

uncommon. In the same time, to facilitate students in learning and communicating their mathematical ideas to other students, they need to learn conventional representation forms. In line with the advance of the use of information and technology, the integration of technology in learning mathematics can potentially enhance students to adapt in the using of new mathematical representations.

Based on the research of Shimizu (2000) and Yamada (2000), it is known that the teacher's role is central in the learning process, which includes: giving encouragement in representation and developing students' thinking process. According to Shimizu (2000), the teacher's questions during learning process can effectively guide students thinking process to right solutions. According to Yamada (2000), questions that posed by the teacher can effectively help students in representation to achieve right solutions.

Therefore, it is needed to conduct a research on mathematical communication and representation, particularly, for Junior high School students. The specific aims of the research are the following:

1. How do the design and forms of mathematics learning materials that can be used to elicit the abilities of mathematical communication and representation of students grade VII?
2. What kinds of mathematical and representation abilities that appear from students grade VII in mathematics learning processes?
3. What factors that influence students' abilities in mathematical communication and representation?
4. How should teacher do to master a series of mathematical communication abilities in teaching-learning mathematics?

The urgency of this research can be classified into three parts. First, theoretically, this research can be used as a basis for developing learning materials, model and learning approaches—that can enhance mathematical communication and representation of Junior High School Students, and bridge other competencies, particularly, mathematics understanding an

problem solving. By investigating students' obstacles in learning mathematics, it can be used as a basis for developing learning approach. Second, the use of model, method, and approach of learning can be more suitable because based on preliminary research—that investigate students' characteristic, teacher's habits, etc. Third, for policy makers, this research can be used as a basis for applying certain policy in enhancing students' ability in mathematics.

METHOD

The research method used in this research is called *a design research* (which is a type of developmental research). According to Gravemeijer & Cobb (2006); Gravemeijer (2004); serta Cobb, et al (2003) in Al Jupri (2008), *design research* consists of three phases: *preliminary design*, *experiment*, dan *retrospective analysis*.

1. Preliminary design

In this phase, we design a hypothetical learning trajectory (HLT). HLT is an anticipation of students' thinking processes before learning-teaching processes. The anticipations include: students abilities both on mathematical communication and representation, and students thinking processes during learning mathematics. To design HLT, we can do by studying relevant literatures, discussion with experienced teachers, the teachers that will be participated in this research, and discussion with the relevant experts—particularly regarding mathematical communication and representation abilities.

According to Simon (1995, in Bakker, 2004), HLT consists of three: learning objectives, learning activities, and a hypothesis of learning-teaching situation. In this first phase, HLT can function as a guide to design learning materials which focus on mathematical communication and representation, class observation, and interviews, etc.

2. Experiment

In this phase, designed learning materials is used in classroom situation. This experiment is aimed to see whetehr what we have predicted is fit with the reality or not. The experiences from this phase can be the basis for redesign the HLT to be new better HLT for next learning processes. The function of HLT in this phase is to guide directly how to do learning processes, observation, and interview.

3. Retrospective Analysis

In this phase, all data is collected from the two previous phases. The processes include: a comparison between HLT and the reality that happen in classroom, analysis of the factors why the HLT is the same like in classroom or not, and synthesize of how to revise the HLT that can be used in the next cycles.

RESULTS AND DISCUSSION

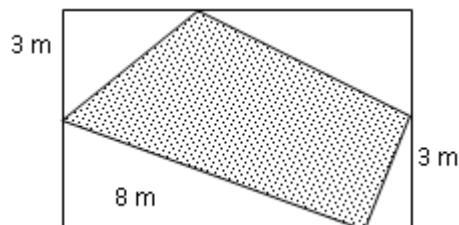
This research is to reveal in what forms the design of learning materials that can be used to reveal students' ability in mathematical communication and representation? What kinds of mathematical and representation abilities that appear from students grade VII in mathematics learning processes? What factors that influence students' abilities in mathematical communication and representation? How should teacher do to master a series of mathematical communication abilities in teaching-learning mathematics?

Based on classroom observations, interviews, discussion with the teacher, analysis of students' works, and questionnaire, we have the following results:

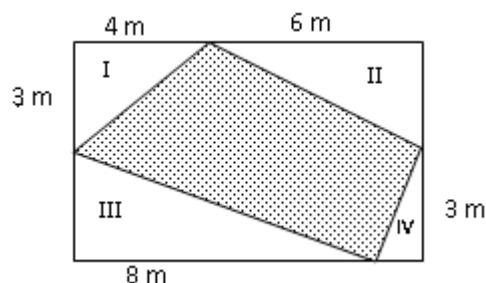
To reveal students' mathematical communication and representation abilities, the learning materials should have the following characteristics: using contextual problems; the given problems should fit with students' experience and "world"; using challenging problems; and using simple students' worksheet.

The mathematical communication abilities that appear from students grade VII are as follows: many students used written expression rather than verbal expression, posing written ideas still converges to final answers, students frequently forget to write down their explanation to solve the problems—except it was asked. For example, when students were asked: "Is it possible if two triangles with different kinds (an obtuse triangle and the other an isosceles triangle) have the same area?" More than half of students answered that it is possible, but they did not give the reasons.

Students would have questions and answered peer questions if it was asked by the teacher. If they have not been able yet to answer questions, they did not directly asked to the teacher, but find out first by themselves. For example, students were given the following problem:



Problem: Uncle has a field in the rectangle form, length 10 m and width 6 m. In each side, there is a guava tree. If the four guava trees are bounded by a rope, then it makes a new quadrangle, could you help finding out the area of the quadrangle? Explain your reasons!



Most of the students solved this problem by numbering the area of right triangles, and finding out its areas. However, since not all sides of the triangles are known, many students used an estimation to predict the sides of the triangles, i.e., the sides of

triangles I and II 6m and 4 m respectively. Therefore, the result is as follows:

$$\begin{aligned} \text{The shaded area} &= \text{Area of the quadrangle} - \text{Area of triangles (I + II + II + IV)} \\ &= (10 \times 6) - \left\{ \left(\frac{1}{2} \times 4 \times 3 \right) + \left(\frac{1}{2} \times 6 \times 3 \right) + \left(\frac{1}{2} \times 8 \times 3 \right) + \left(\frac{1}{2} \times 2 \times 3 \right) \right\} \text{ cm}^2 \\ &= 60 - (6 + 9 + 12 + 3) \\ &= 60 - 30 \text{ m}^2 \\ &= 30 \text{ m}^2 \end{aligned}$$

Find the area of the shaded area if the sides are 7m dan 3m? The result is still the same:

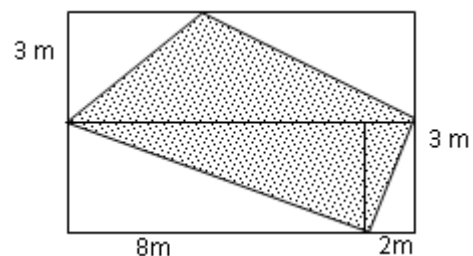
$$\begin{aligned} \text{Luas yang diarsir} &= \text{Luas segiempat} - \text{Luas segitiga (I + II + II + IV)} \\ &= (10 \times 6) - \left\{ \left(\frac{1}{2} \times 3 \times 3 \right) + \left(\frac{1}{2} \times 7 \times 3 \right) + \left(\frac{1}{2} \times 8 \times 3 \right) + \left(\frac{1}{2} \times 2 \times 3 \right) \right\} \text{ m}^2 \\ &= 60 - (4,5 + 10,5 + 12 + 3) \\ &= 60 - 30 \text{ m}^2 \\ &= 30 \text{ m}^2 \end{aligned}$$

Based on the above situation, what is your conclusion? Indeed, the triangles I and II should be enough if only have one size, since the sum of the areas is the same.

However, using the given situation, many students were in confusion because they were not able to predict the area of each triangle without predicting one of its sided. This condition made students were not sure with their answers. This situation is actually the same as the teacher's prediction which stated that: if the data in the problem is unclear, students would ask, for instance, what is the size of the above side? Of course, to answer this question, the teacher did not given the information directly, but the teacher suggested the students to make a prediction to come to the right answer. For example, a student proposed one way to find out the shaded area

by calculating directly. The teacher asked two students, having different ways, to explain it.

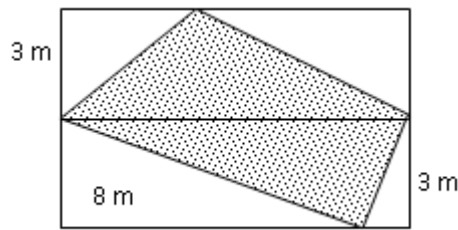
One of the two students divided the shaded area into three areas, as follows:



Why should you divide the area into three? He answered, because the sides are known. From this answer, it seems that the student and maybe most of other students stick only on the size of the sides. Therefore, the area is as follows:

$$\begin{aligned} \text{The shaded area} &= \text{Area of triangles (I + II + III)} \\ &= \left(\frac{1}{2} \times 10 \times 3 \right) + \left(\frac{1}{2} \times 8 \times 3 \right) + \left(\frac{1}{2} \times 2 \times 3 \right) \text{ cm}^2 \\ &= 15 + 12 + 3 \\ &= 30 \text{ m}^2 \end{aligned}$$

Other student divides the shaded area into two:



Which, therefore, the area is as follows:

$$\begin{aligned}
 \text{Luas yang diarsir} &= \text{Luas segitiga (I + II)} \\
 &= \left(\frac{1}{2} \times 10 \times 3\right) + \left(\frac{1}{2} \times 10 \times 3\right) \text{ m}^2 \\
 &= 15 + 15 \\
 &= 30 \text{ m}^2
 \end{aligned}$$

When known that both areas are the same, several students raised up their hands, and made a the following conclusion:

Student 1 : To find out the area of the shaded area can be calculated directly.

Teacher : Do you agree with your friend conclusion, students?

Students : (All students agree)

Teacher : Is there any other conclusion?

Student 2 : There are several ways to find the area of the shaded area.

Teacher : Yes, that is right! Are there still other ways?

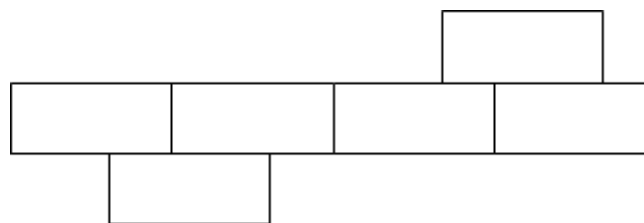
Student 3 : Yes there is, “ calculating triangles determined only by

knowing its base and height, not based on its forms.

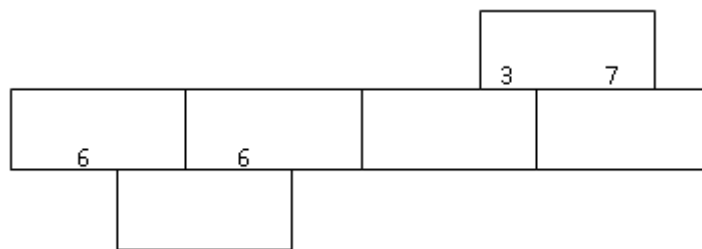
Based on the three conclusions above, the Student 3 ‘c conclusion is better than the others—which means he is better in representing mathematical concepts.

In general, students used illustrations arithmetically rather than algebraically or geometrically. For example, when students were given the following problem:

A blueprint of a building can be drawn by several rectangles. If each rectangle has 10 long and 5 m wide, find the perimeter of the building below.



Majority of students made a prediction of the sides of the rectangles, as the following:

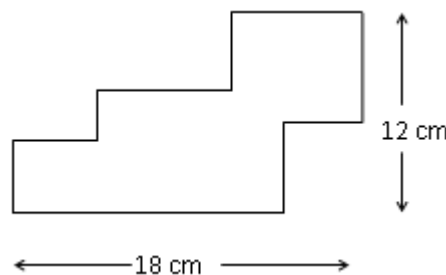
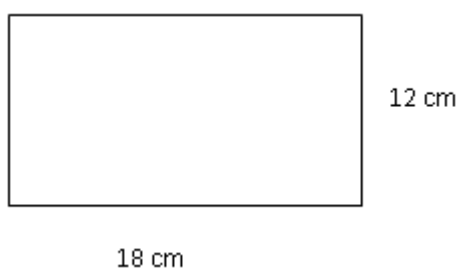


When they are asked, why did you place 3 and 7? Is it based on your measuring? Could we place 2 and 9? Many students gave reasons that 3 and 7 are the most reasonable measures. Other students argued that other measures also possible, if the sum is 10.

Could you express in more general to make the fix sum is satisfied? Majority of students

could not be able to explain it, only few students were able express the measures into a and b . The same situation also happened, when students were given the following problem

Blueprint of two buildings are given as follows. Which one has longer perimeter? Explain your answer!



In general, students predicted the length of each segment of the right figure. Other students measured using rulers to find the length each segment of the right figure. Only few students stated that the length of segments is constant although the figure changed. And, very few students were able to represent each segment using algebraic symbols. These results show that students' external representation is still not spread to all students.

The following are factors from the teacher that influence students' mathematical communication and representation of grade VII in learning mathematics. The teacher should be able to present learning activities, communicate mathematical ideas to students, construct mathematical problems, give proper intervention and help to students, use collaborative approach, make didactic-pedagogic situations, etc.

Mathematical representation abilities from students of grade VII (Junior high school students) are the following: Internal representation can not be expressed yet

externally. Students seem find difficulties to represent externally. This can be seen, when students were given the problem: "A right triangle with the size 8m x 6 m, students are asked to create new triangle which has area twice than the origin triangle. One student said, "It is easy, I can multiply it by two." When he was asked to represent his idea, he found difficulties to represent his ideas. This also is the same as other students. Students who had known the answer could not be able to write their ideas. And several students said, "We are still thinking how to write our ideas."

In using representations to solve problems, even students have known that the problems can be solved using enough data, students were still not realize yet what they should do. For example, given the following problem,

There are three triangles. The area of the first triangle is equal to twice of the second triangle. And the area of the second triangle is equal to twice the third triangle. What can you

conclude if the heights of the triangles are the same? What can you conclude if the bases of the triangles are the same?

In general, students can make a model/representation from the given problem, although they draw right triangles. Only few students used tables to make a conclusion, whereas, other students only used a single data to make a conclusion.

Several efforts that were used by the teacher to master a series of mathematical communication are: making better plan, creating HLT (Hypothetical Learning Trajectory) accurately, giving proper intervention to students, creating cognitive-conflict, and balancing communication aspect in learning processes.

CONCLUSION

Based on the results of this research we can draw on the following conclusion:

1. To develop mathematical communication and representation abilities we need: (i) learning materials that fit with students' thinking "world", (ii) use contextual problems to stimulate students' learning, (iii) teacher intervention properly to help building on external representation and to proficient students' mathematical communication.
2. Problem-Based Learning can be used as an alternative approach to be considered, particularly on the teacher's intervention model during learning-teaching process.
3. Based on the analysis of students' works, we found that there are several difficult indicators of mathematical communication and representation that are difficult to be developed, therefore, needed the teacher's interventions. For example, communicating mathematical ideas both verbally and algebraically, using representation for solving problems, posing argumentations to find patterns.
4. Based on observation, it is known that the teacher found difficulties in analyzing

students' mathematical communication during learning-teaching process.

5. Students' difficulties in understanding mathematics may also be the teacher's difficulties in giving intervention during learning processes. Therefore, the teacher should be able to predict students' difficulties even before learning-teaching processes happen. In addition, the teacher also should be able to choose the proper intervention to help students.

REFERENCES

- Al Jupri, Yulianti, K., Rukmana, K., Saputra, C. (2007). *Pengembangan Desain Pembelajaran Matematika Realistik untuk Menumbuhkembangkan Kemampuan Pemecahan Masalah dan Komunikasi Matematis Siswa Kelas VIII H SMP 22 Bandung*. Bandung: Laporan Penelitian (Tidak dipublikasikan)
- Al Jupri. (2008). *Computational Estimation in Grade Four and Five: Design Research in Indonesia*. Utrecht: Master Thesis at the Freudenthal Institute, Utrecht University (Unpublished).
- Ansari, B. I. (2003). *Menumbuhkembangkan Kemampuan Pemahaman dan Komunikasi Matematik Siswa SMU melalui Strategi Think-Talk-Write*. Disertasi pada Program Pascasarjana UPI, Bandung: Tidak dipublikasikan.
- As'ari, A.R. (2001). *Representasi: Pentingnya dalam Pembelajaran Matematika*. Jurnal MATEMATIKA, TAHUN VII, Nomor 2, Agustus 2001.
- Bakker, A. (2004). *Design research in statistics education: On symbolizing and computer tools*. Utrecht: Freudenthal Institute.
- Cobb, P., Confrey, J., diSessa, A. A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, 32(1), 9–13.
- Cobb, P. & Bauersfeld, H. (Eds.) (1995). *The Emergence of Mathematical*

- Meaning: Interaction in classroom cultures. Hillsdale, New Jersey: Lawrence Erlbaum Associates, publishers.
- Depdiknas. (2006). *Kurikulum Tingkat Satuan Pendidikan*. Jakarta:
- Dekker, R. dan Moher, M.E.(2004). Teacher Interventions Aimed at Mathematical Level Raising during Collaborative Learning. *Education Studies in Mathematics 56*: 39-65. Netherlands: Kluwer Academic Publishers.
- Delisle, R. (1997). *How to Use Problem Based Learning in the Calssroom*. New York: Association for Supervision and Curriculum Development.
- Downs, J.M. dan Downs,M.(2002). Advanced Mathematical Thinking with a Special Reference to Reflection on Mathematical Structure. Dalam *Handbook of International Research in Mathematics Education*. English, L.D. (Ed). NCTM. London: Lawrence Erlbaum Associates Publisher.
- Eisner, E. W. (1997). Cognition and Representation: A Way to Pursue the American Dream. *Phi Delta Kappa*, 349-353.
- Emori, H. (2005). *Constructing Original Mathematics Educationfor Keeping Thai Ethnic Identity*. Panduan Workshop. Gunma University. Tidak diterbitkan.
- Esty, W.W. & Teppo, A.R. (1996). Algebraic Thinking, Language, and Word Problems. In P.C. Elliot & M.J. Kenney (Eds). *Communication in Mathematics, K-12 and Benyond* (1996 Yearbook). Virginia: NCTM.
- Fogarty, R. (1997). *Problem Based-Learning and the Other Curriculum Models for Multiple Intelegences Clasroom*. Hawker Brownlow Education.
- Gijsselaers, W. H.(1996). Connecting Problem-Based Practice with Educational Theory. Dalam Wilkerson, L.(Ed). *New Direction for Teaching and Learning*. No.68. Josey-Bass Publishers.
- Goldin ,G.A. (2003). Representation in Mathematical Learning and Problem Solving. Dalam English, L.D. (Ed). *Hanbook of International Research in Mathematics Education*. NCTM. London: Lawrence Erlbaum Associates, Publisher.
- Gravemeijer, K.P.E. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD- β Press. Freudenthal Institut.
- Greenes, C. & Schulman, L. (1996). Communication Processes in Mathematical Explorations and Investigation. In P.C. Elliot & M.J. Kenney (Eds). *Communication in Mathematics, K-12 and Benyond* (1996 Yearbook). Virginia: NCTM.
- Grouws, D.A. (1992). *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillan Publishers.
- Hendayana, S., et.al (2007). *Lesson study: Suatu Strategi untuk Meningkatkan Keprofesionalan Pendidik (Pengalaman IMSTEP-JICA)*. Bandung: UPI Press.
- House, P.A.(1996).Try a Little of the Write Stuff. Dalam Elliot, P.C. dan Kenney,M.J. (Eds). *Communication in Mathematics,K-12 and Beyond..* Yearbook Virginia: NCTM.
- Honebein, P. C. (1996). Seven Goals for the Design of Constructivist Learning Environments. Dalam Wilson, B.C. (Ed.), *Constructivist Learning Environments: Case Studies in Instructional Design* (pp. 11-24). Englewood Cliffs, NJ:Educational Technology Publications.
- Howey, K.R., et.al (2001).*Contextual Teaching and Learning:Preparing Teacher to Enhance Student Success in The Work Place and Beyond*.Washington: ERIC Clearinghouse on Teaching and Teacher Education.
- Huinker, D., & Laughlin, C. (1996). Talk Your Way into Writing. In P.C. Elliot

- & M.J. Kenney (Eds). *Communication in Mathematics, K-12 and Beyond* (1996 Yearbook). Virginia: NCTM.
- Lester, F.K. (1980). Research on Mathematical Problem Solving. In R.J. Shumway (Ed.), *Research in Mathematics Education.* (pp. 286-323). Reston, Virginia: National Council of Teachers of Mathematics.
- Linquist, M.M. (1996). Communication an Imperative for Change: A Conversation with Mary Lindquist. In P.C. Elliot & M.J. Kenney (Eds). *Communication in Mathematics, K-12 and Beyond* (1996 Yearbook). Virginia: NCTM.
- Massingila, J.O. dan Wistiniowska, E.P. (1996). Developing and Assessing Mathematical Understanding in Calculus Through Writing. Dalam Elliot, P.C. dan Kenney, M.J. (Eds). *Communication in Mathematics, K-12 and Beyond.* Yearbook Virginia: NCTM.
- McCoy, L.P., Baker, T.H., & Little, L.S., (1996). Using Multiple Representation to Communicate: An Algebra Challenge. In P.C. Elliot & M.J. Kenney (Eds). *Communication in Mathematics, K-12 and Beyond* (1996 Yearbook). Virginia: NCTM.
- Monk, S. (2003). Representation in School Mathematics: Learning to Graph and Graphing to Learn. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp.250-262). Reston, NJ: NCTM.
- NCTM (National Council of Teacher of Mathematics). (1989). *Curriculum and Evaluation Standard for School Mathematics.* Reston, Va: NCTM.
- NCTM. (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va: NCTM.
- NCTM. (2000). *Principles and Standadrs for School mathematics.* Reston, VA:NCTM
- Ngeow, K.K., dan San, Y. (1997). *Learning to Learn: Preparing Teachers and Students for Problem-Based Learning.*[On-Line], Tersedia: <http://eric Indiana.Edu>. ERIC Clearinghouse on Reading English and Communication Bloomington. IN.
- Pierce, J.W. dan Jones, R.T. (2001). Problem-Based Learning: Learning and Teaching in Context of Problems. Dalam K.R. Howey, dkk. (Eds). *Contextual Teaching and Learning: Preparing Teacher to Enhance Student Success in the Work Place and Beyond.* (pp. 17-32). ERIC Clearinghouse on Teaching and Teacher Education.
- Pugalee, D.K. (2004). A Comparison of Verbal and Written Description of Students' Problem Solving Process. *Educational Studies in Mathematics*, 55: 22-47. Kluwer Academic Publishers.
- Reidesel, C.A., Scwartz, J.E., & Clements, D.H. (1996). *Teaching Elementary School Mathematics.* Boston: Allyn and Bacon.
- Sabandar, J. (2005). *Pertanyaan Tantangan dalam Memunculkan Berpikir Kritis dan Kreatif dalam Pembelajaran Matematika.* Makalah Disajikan pada Seminar MIPA di JICA: tidak diterbitkan.
- Savery J.R. dan Duffy, T.M. (1996). Problem -Based Learning: An Instructional Model and Its Constructivist Framwork. .[On-Line], Tersedia: <http://www.Soe.ecu.edu/Itidi/colaric/KB/PBLs.htm>
- Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education.* 26 (2), 114-145.
- Shigeo, K. (2000). On Teaching Mathematical Thinking. In O.Toshio (Ed.), *Mathematical Education in Japan* (pp. 26-28). Japan: JSME.
- Shimizu, N. (2000). An Analysis of "Make an Organized List" Strategy in Problem Solving Process. In T. Nakahara & M.

- Koyama (Eds.) *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4 (pp. 145-152). Hiroshima: Hiroshima University.
- Siegel, M., Barosi, R., Fonzi, J.M., & Sanridge, L.G. (1996). Using Reading to Construct Mathematical Meaning. In P.C. Elliot & M.J. Kenney (Eds). *Communication in Mathematics, K-12 and Beyond* (1996 Yearbook). Virginia: NCTM.
- Supriadi, D. (2001). *Anatomi Buku Sekolah di Indonesia*. Yogyakarta: Adi Cita
- Smith, S. P. (2003). Representation in School Mathematics: Children's Representations of Problems. Dalam J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics* (pp. 263-274). Reston, NJ: NCTM.
- Sumarmo, U.(2002). *Alternatif Pembelajaran Matematika dalam Menerapkan Kurikulum Berbasis Kompetensi*. Makalah pada Seminar Nasional. FPMIPA UPI: Tidak dipublikasikan.
- Shigeo, K. (2000). On Teaching Mathematical Thinking. In O.Toshio (Ed.), *Mathematical Education in Japan* (pp. 26-28). Japan: JSME.
- Shimizu, N. (2000). An Analysis of "Make an Organized List" Strategy in Problem Solving Process. In T. Nakahara & M. Koyama (Eds.) *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4 (pp. 145-152). Hiroshima: Hiroshima University.
- Usiskin, Z. (1996). Mathematics as a Language. In P.C. Elliot & M.J. Kenney (Eds). *Communication in Mathematics, K-12 and Beyond* (1996 Yearbook). Virginia: NCTM.
- Wood,T. (1999).Creating a Context for Argument in Mathematics Class. *Journal for Reasearch in Mathematics Education*, 30 (2). 171-191.
- Yamada, A. (2000). Two Patterns of Progress of Problem-Solving Process: From a Representational Perspective. In T. Nakahara & M. Koyama (Eds.) *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4 (pp. 289-296). Hiroshima: Hiroshima University.