AN ANALYSIS OF NUMBER SENSE AND MENTAL COMPUTATION IN THE LEARNING OF MATHEMATICS

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Abstract. The purpose of this research was to assess students' understanding of number sense and mental computation among Form One, Form Two, Form Three and Form Four students. A total of 1756 students, ages ranging from 12 to 17 years, from thirteen schools in Selangor participated in this study. A majority (74.9%) of these students obtained an A grade for their respective year-end school examinations. The design for this study was quantitative in nature where the data on student's sense of numbers was collected using two instruments, namely, Number Sense Test and Mental Computation Test. Each of these instruments consisted of 50 and 45 items respectively. The results from this study indicate that students were not able to cope to the Number Sense Test as compared to the Mental Computation Test. The former unveils a low percentage of 37.3% to 47.7% as compared to the latter of 79% to 88.6% across the levels. In the number Sense Test, surprisingly, there was no significant difference in the results between Form 1 students and Form 2 students and also between Form 3 students and Form 4 students. This seems to indicate that as the number of years in schools increase, there is an increasing reliance on algorithm and procedures. Although in the literature it has been argued that including mental computation in a mathematics curriculum promotes number sense (McIntosh et. al., 1997; Reys, Reys, Nohda, & Emori, 2005), this was not the case in this study. It seems that an over reliance on paper and pencil computation at the expense of intuitive understanding of numbers is taking place among these students.

BACKGROUND

Learning what numbers mean, how they may be represented, relationships among them and computations with them are central to developing number sense (Munirah, 2002; Munirah, Rohana Alias, Noor Asrul & Ayminsyadora, 2010). Number sense refers to a person's general understanding of numbers and operations along with the ability to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for solving complex problems (Burton, 2003; Reys, 2001). Researchers note that number sense develops gradually, and varies as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms (Howden, 1989).

"Number sense develops over time and the development is best if the focus is consistent, day by day, and occurs frequently within each mathematics lesson" (Thornton & Tucker, 1989, p. 21). Number sense is more of a way of teaching than a topic to be taught" (Van de Walle and Watkins, 1993). As a foundation for secondary level, the Ministry of Education of Malaysia requires students to master the 3R's (which includes arithmetic) during the primary level. The Malaysian primary school mathematics curriculum clearly states that the main aspect of the curriculum is to build and develop children's understanding in the number concept and at the same time attain high facility in the basic skill. This understanding of numbers is of fundamental importance and is basically the main ingredient in problem solving situations. This will be beneficial in helping them through their secondary education curriculum that is based on three strands namely number, shape and relation (Kementerian Pendidikan Malaysia, 1989, Kementerian Pendidikan Malaysia 2003). It encourages them to be problem solvers in the context of practical situations and helps them in making sense of what goes on around them. It will also promote the idea that mathematics is a discipline that requires logical thinking, which can lead to better knowledge enhancement in other disciplines. Based on the National Education policy, the mathematics curriculum in KBSM is planned to provide students with experiences that may consist of the:

a) integration of knowledge, values and language;
b) integration of mathematics with other branches of knowledge;
c) integration of various topics in mathematics;
d) integration of mathematics learned in classrooms with those experiences outside the classrooms (Kementerian Pendidikan, 2003).

This KBSM curriculum very much relates to the integration of thinking and numbers in mathematics teaching and learning. This complex, multifaceted and dispositional nature of integration suggests that it cannot be confined to specific textbook chapters. Rather, the development of thinking complexities and number sense results from a whole range of activities that permeates the entire approach to the teaching of mathematics (Greeno, 2001). We believe that understanding numbers becomes more essential especially when they proceed to secondary school and the question that arises is that, "Have Malaysian school children in lower secondary mastered number sense well enough so as to be able to grasp the content of secondary school syllabus as vision in KBSM? ".

OBJECTIVE OF STUDY

The improvement of mathematics education for all students requires effective mathematics teaching in all classrooms. Determining what experience might be important to foster this understanding requires a thorough analysis of a student's number sense in various mathematical concepts. In a nutshell, the objective of this study is to

i. gauge students' understanding in number sense of basic mathematical concepts,
ii. assess whether children demonstrate understanding of numerical situations in which they solve number problems.

These students have been formally taught these fundamental mathematical concepts and this research will enable us to assess these students' sense of numbers. It will give us an insight into whether these students have number sense abilities or are just performing algorithmic procedures to get answers without taking into consideration if the answers make sense.

RESEARCH METHODOLOGY

The methodology that was utilized in this study encompassed the quantitative method where the data provided a bearing on how students respond to a given set of problem tasks in Number Sense.

Subjects. The subjects for this study comprised 1756 students from the levels of Form One, Form Two, Form Three and Form Four (ages ranged from 13 to 16 years old) from 13 schools in the state of Selangor. The rationale for choosing levels of Form 1 and Form 2 was that students at this level have been formally taught the basic mathematical concepts needed to solve problems during their primary school days (first six years). The Form 3 and Form 4 levels were selected to assess these students' sense of numbers after 10 years of formal education and also for the purpose of comparison with the lower secondary students.

Instrument and Administration of the Instrument. All students were given a 50-item paper and pencil test on number sense. The test items were adapted from a number sense test published by McIntosh (McIntosh et al., 1997), which comprised five number sense strands in their framework:

1. Understanding and use of the meaning and size of numbers.
2. Understanding and use of equivalent forms and representations of numbers.
3. Understanding the meaning and effect of operations.
4. Understanding and use of equivalent expressions.
5. Computing and counting strategies.

ANALYSIS AND RESULTS

As indicated in table 1, the composition of the samples are 31.7 % in Form 1, 33.5% in

Form 2, 14.2% in Form 3 and 20.7% in Form 4.

Of the 1660 responses received for mathematics grade obtained in their school exam, the majority (74.9%) obtained an A grade for their respective year-end examinations. This was followed by 20.5%, 4.5% and 0.2% respectively for grades B, C and D. Table 2 details the breakdown of the grades obtained by students according to their levels.

Table 1. Demographics of respondents by level

Level	Frequency	Percent
Form 1	556	31.7
Form 2	588	33.5
Form 3	249	14.2
Form 4	363	20.7
Total	1756	100.0

Table 2. Demographics of respondents by examination grades

Grade	Frequency	Percent
A	1243	70.8
B	340	19.4
C	74	4.2
D	3	.2
Total	1660	94.5
Missing	96	5.5
Total	1756	100.0

1. Analysis of Number Sense Test

The following sections detail the findings of students' performance based on the Number Sense Test and Mental Computation Test.

2. Analysis of Number Sense Test Across Levels

On each of the test items in the Number Sense test, a score of one is given for a correct answer while a zero score is awarded for an incorrect answer.

Table 3. Summary Statistics for Number Sense Test by Levels

Level	N	Mean (Max. 50)	Percentage Correct	Std. Deviation
Form 1	556	18.6457	(37.3%)	6.01338
Form 2	588	19.3112	(38.6%)	7.05805
Form 3	249	23.3655	(46.7%)	7.05805
Form 4	363	23.8292	(47.7%)	6.57750

Table 3 shows that the mean score on the test increases with age. The percentage of average score for the Number Sense test is less than 50% across all levels. The lowest percentage of average score on this test is 37.3% (Form 1) and the highest is 47.7% (Form 4). The increase is the most from Form 2 (38.6%) to Form 3 (46.7%).

3. Difference in Mean Score for Number Sense Test

A review of Table 3 showed that there was a difference in the mean score between levels in the Number Sense test. In order to analyze whether the mean difference was statistically significant, an F test was done as shown in table 4.

Table 4. Comparison of Means between Levels in Number Sense Test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	8789.608	3	2929.869	69.034	0.000
Within Groups	74356.399	1752	42.441		
Total	83146.007	1755			

The F test to compare mean scores on the Number Sense test between levels indicates a significant difference with an F-value of 69.034 (p-value 0.000) as shown in the table 4.

Multiple comparisons using LSD, Scheffe's and Duncan's multiple comparisons tests as shown in table 5, indicates that there is a significant difference in mean score on the Number Sense test at the 0.05 level between students of all levels except between Form 1 and Form 2 students as well as between Form 3 and Form 4 students.

Table 5. Multiple Comparisons of Means between Levels on Number Sense

Dependent Variable	Form (I)	Form (J)	Mean Difference (I – J)	Std. Error	Sig.
Number Sense Test	1	2	-0.66554	0.38537	0.084
		3	-4.71978*	0.49677	0.000
		4	-5.18352*	0.43960	0.000
	2	1	0.66554	0.38537	0.084
		3	-4.05424*	0.49257	0.000
		4	-4.51798*	0.43485	0.000
	3	1	4.71978*	0.49677	0.000
		2	4.05424*	0.49257	0.000
		4	-0.46374	0.53606	0.387

* The mean difference is significant at the .05 level

4. Analysis of Mental Computation Test

There were 45 items in the Mental Computation test and the score assignment is similar to the Number Sense test. A score of one is given for a correct response while a zero score is awarded for an incorrect response. As such, the total score for the Mental Computation test is 45 respectively. Table 6 displays the score obtained by students across the four levels.

The Test score ranged from a mean of 35.9 to 39.1 with the correct percentage response ranging from 79.9% to 86.8%. There is an increase in average score on the Mental Computation test in transition from Form 2 to Form 3. Surprisingly on this test, there is a small drop from Form 1 (79.9%) to Form 2 (79%) and also from Form 3 (88.6%) to Form 4 (86.8%).

Table 6. Summary Statistics for Mental Computation Test by Levels

Level	N	Mean (Max. 45)	Std. Deviation
Form 1	556	35.9388 (79.9%)	6.60165
Form 2	588	35.5561 (79.0%)	7.38468
Form 3	249	39.8876 (88.6%)	5.06922
Form 4	363	39.0579 (86.8%)	4.75992

5. Difference in Mean Score for Mental Computation Test

As shown in table 6, there is a difference in the mean score for the Mental Computation test between levels. To analyze these differences, an F test was conducted as shown in table 7. The results show that there is a significant difference in the mean score between levels in the Mental Computation test with an F-value of 45.094 (p-value 0.000).

Table 7. Comparison of Means between Levels in Mental Computation Test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	5464.879	3	1821.626	45.094	0.000
Within Groups	70773.705	1752	40.396		
Total	76238.585	1755			

To determine the difference between the levels, multiple comparisons tests using LSD, Scheffe's and Duncan's were conducted. The results revealed that the same significant differences that were prevalent in the Number Sense test were also evident in the Mental Computation test as shown in table 8. In other words, there are significant differences at the 0.05 level of significance between all levels except between Form 1 and Form 2 students as well as between Form 3 and Form 4 students on the Mental Computation test.

Table 8. Multiple Comparisons of Means between Levels on Mental Computation

Dependent Variable	Form (I)	Form (J)	Mean Difference (I – J)	Std. Error	Sig.
Mental Computation Test	1	2	0.38273	0.37597	0.309
		3	-3.94870*	0.48465	0.000
		4	-3.11900*	0.42888	0.000
	2	1	-0.38273	0.37597	0.309
		3	-4.33143*	0.48056	0.000
		4	-3.50173*	0.42425	0.000
	3	1	3.94870*	0.48465	0.000
		2	4.33143*	0.48056	0.000
		4	0.82970	0.52299	0.113

DISCUSSION AND CONCLUSION

The results from this study indicate that the students at all levels namely Form 1, Form 2, Form 3 and Form 4 were not able with cope to the Number Sense Test where it unveils a low mean score ranging from 18.6 to 23.8 with a maximum score of 50 and a low correct percentage of 37.3% to 47.7% across the levels. In other words, these students' received a score of less than 50% achievement in the Number Sense Test. In the mental Computation Test, the mean score obtained was in the high range of 35.5 to 39.9 (max score of 45) for the Form 1, Form 2, and Form 3 and Form 4 students respectively. In terms of percentage of correct responses, the score ranged from 79% to 88.6%.

Surprisingly, quite to the contrary of one's expectation, there was no significant difference in the mean score between Form 1 students and Form 2 students and also between Form 3 students and Form 4 students in both the Number Sense test and Mental Computation test. This seems to indicate that as the number of years in schools increase, there is no increase in knowledge and maturity usually associated with the experience one gets as one goes to the higher level of education. Perhaps the poorer results of Form 2 students compared to Form 1 students and Form 4 students compared to Form 3 students on this test could be explained by an over-reliance on algorithm and procedures.

It is said that mental computation promotes greater understanding of the structure of number and its properties (Reys, 2001); can be used as a "vehicle for promoting thinking, conjecturing, and generalizing based on conceptual understanding" (Reys & Barger, 1994, p. 31) and it is also explicitly stated that mental computation promotes number sense (National Council of Teachers of Mathematics, 1989; Sowder, 1990). However in this study, the percentage of difference in student's performance in the two tests shows a vast disparity. It seems to indicate that the focus of teaching and learning in the classroom has been on Mental computation in the expense of Number Sense if we compare the performance of students in this two tests. The former yields a percentage of about 42 % as compared to the latter, a high 85% and this finding does not seem to support the claim (NCTM, 1989; Sowder, 2009; Reys, 2001) that mental computation promotes number sense. This is a call for alarm because we believe that although both are important, the number sense should prevail the latter.

The findings on students' achievement in the Number Sense Test seems to indicate an existence of a gap between the ability to do paper-and-pencil calculations and intuitive understanding. Research (Munirah, et. al, 2010; Parmjit, 2008; Parmjit, 2001) has indicated that Malaysian students are good at computational skills and once they understand these procedures, practice will help them become confident and competent in using them. However, research indicates that if students memorize mathematical procedures without understanding, it is difficult for them to go back later and build intuitive understanding (Resnick and Omanson 2007; Wearne and Hiebert 2008). When students memorize without understanding, they may confuse methods or forget steps (Kamii and Dominick 2008) and we believe that is the scenario among students of this study.

Majority of the students (74.9%) involved in this study obtained an A grade for their examination. However, there is a vast disparity between the grade score and the Number Sense test where students' sense of numbers is very much lacking across all levels. The probable reason for this is the inadequate mathematical instructions in schools and this result in many children having inadequate understanding of number sense of mathematical concepts.

The results reported in this study are a cause for concern. Given that a number of items have been answered worse by students as their school experience increases especially when there is no statistical difference between Form1 and Form 2 students and similarly, between Form 3 and Form 4 students, one has to ask whether teachers are unwittingly causing children to learn math through a reliance on drill and practice, without the profound understanding of fundamental mathematics (PUFM) suggested by Ma (1999). If this is the case, we may be preventing students getting a "greater facility with analyzing and making sense of data and deeper conceptual understanding of mathematics,"(p.306, Olson & Berk, 2007). Even though the national mathematics Curriculum (1989) suggests that teachers need to emphasize number sense, teachers seem unwilling to let go of the adage that "practice makes perfect," with practicing algorithms made synonymous with "practice."

We believed that the current practice of the Ministry that emphasizes algorithmic mastery in arithmetic learning in schools is misguided. The students that are being "trained" under this environment fail to develop an understanding of the underlying mathematics, and in fact soon lose their grasp on the very skills that were intended to be the focus of their education. To address this problem we propose changes in direction and emphasis, in both curriculum and pedagogy. These changes are often presented as a way to help students develop number sense that will eventually have a positive development in the learning of mathematics. To answer the One million Dollar question, *"how can number sense be developed?",* Greeno (2001) suggests that "it may be more fruitful to view number sense as a by-product of other learning than as a goal of direct instruction" (p. 173). Howden (1989) expresses the view that number sense "develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (p. 11). We will conclude that the development of number sense requires an environment that fosters curiosity and exploration at all grade levels.

Attention to number sense when teaching a wide variety of mathematical topics tends to enhance the depth of student ability in this area. Competence in the many aspects of number sense is an important mathematical outcome for students. Over 90% of the computation done outside the classroom is done without pencil and paper, using mental computation, estimation or a calculator. However, in many classrooms, efforts to instill number sense are given insufficient attention. As teachers develop strategies to teach number sense, they should strongly consider moving beyond a unit-skills approach (i.e. a focus on single skills in isolation) to a more integrated approach that encourages the development of number sense in all classroom activities, from the development of computational procedures to mathematical problem solving. Although more research is needed, an integrated approach to number sense will be likely to result not only in greater number sense but also in other equally important outcomes.

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