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THE DEVELOPMENT OF HYPOTHETICAL LEARNING TRAJECTORY FOR LINEAR EQUATIONS WITH PISA AND SCIENTIFIC APPROACH MODEL CONSIDERATION

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ABSTRACT

Prediction of how learning might proceed can be used as a basis for designing and achieving successful learning. A Hypothetical Learning Trajectory (HLT) consisted of learning goals, a set of learning activities, and a hypothesized learning process was developed for learning linear equations in which PISA and the scientific approach model also became points of consideration. HLT implementation in three learning schemes suggested the importance of scaffolding in learning linear equations. Sufficient and strategic scaffolding can improve student's understanding and facilitate the students in overcoming obstacles when learning linear equations.

ABSTRAK

Prediksi tentang bagaimana pembelajaran mungkin berjalan dapat digunakan sebagai dasar untuk merancang dan mencapai pembelajaran yang berhasil. Sebuah *Hypothetical Learning Trajectory* (HLT) yang terdiri dari tujuan pembelajaran, seperangkat kegiatan pembelajaran, dan hipotesis proses pembelajaran dikembangkan untuk pembelajaran persamaan linier dengan PISA dan model pendekatan saintifik juga menjadi bahan pertimbangan. Hasil dari penerapan HLT pada tiga skema pembelajaran menunjukkan pentingnya *scaffolding* dalam pembelajaran persamaan linier. *Scaffolding* yang memadai dan strategis dapat meningkatkan pemahaman siswa dan memfasilitasi siswa dalam mengatasi hambatan ketika mempelajari persamaan linear.

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INTRODUCTION

Learning path is characterized by an expected tendency in which although students often progress along a similar path, individual learning proceeds along an idiosyncratic path (Simon, 1995). Therefore, predicting the path which learning might proceed or what Simon (1995) refers to as Hypothetical Learning Trajectory, will supports teachers with a basis in conducting learning activity. Experts (Gravemeijer, Bowers, and Stephan, 2003; Cobb and Gravemeijer, 2008), supported the positive outlook and benefits of Hypothetical Learning Trajectory (HLT) for learning. Gravemeijer et al. (2003) summarize that HLT view (1) learning trajectory as a socially situated, (2) planning as an iterative cycle rather than a single-shot methodology, (3) students' constructions rather than mathematical content as the main focus of learning, as well as (4) providing teacher a grounded theory on how a specific set of instructional activities might play out in a given social setting.

A Hypothetical Learning Trajectory (HLT) consists of learning goals, a set of learning tasks, and a hypothesized learning process (see Simon, 1995; Simon and Tzur, 2004; Simon, 2014). If learning is a journey, the trajectory is the path you travel, whereas hypothetical trajectory is the path you anticipate at any point (Simon, 1995). By generating the HLT, the selection of learning tasks is not a matter of intuition or trial and error but rather through a systematic mechanism of thinking on how the task can promote the learning process (Simon and Tzur, 2004) because the trajectory of students learning is affected by the opportunities and constraints provided by the structure and content of mathematics lessons (Simon, 2014). Since its introduction (Simon, 1995), HLT has proven to be a beneficial construct in numerous mathematics classes across students' age, education levels, and concepts (Clements, Wilson, and Sarama, 2004;

Lee, 2005; Clements, Sarama, Spitler, Lange, and Wolfe 2011; Sarama, Clements, Barrett, Van Dine, and McDonel, 2011; Blanton, Brizuela, Gardiner, Sawrey, and Newman-Owens, 2015; Ellis, Ozgur, Kulow, Dogan, and Amidon, 2016; Andrews-Larson, Wawro, and Zandieh, 2017; Ivars, Fernandez, Llinares, and Choy, 2018; Clements, Sarama, Baroody, Joswick, and Wolfe, 2019).

In elaborating the use of HLT, Simon and Tzur (2004) highlighted that the most important use of the HLT would be for teaching complex or problematic concepts. The linear equation is a concept that students found problematic in which Huntley, Marcus, Kahan, and Miller (2007) found that students' knowledge flexibility of linear equations is superficial. A previous study by Birinci, Delice, and Aydin (2014) also suggested that even university students still found linear equations difficult. Recently, Adu, Assuah, and Asiedu-Addo (2015) found that students have difficulties in solving linear equations word problems, with only 2% of students arriving at the correct answer. Although HLT has been used as the primary construct for learning mathematics concepts such as geometry (Clements et al., 2004; Clements et al., 2019), measurement (Sarama et al., 2011), functions (Blanton et al., 2015), or exponential growth (Ellis et al., 2016), HLT still rarely used as a construct for linear equations. Therefore, this current article will report the development of HLT as a construct for teaching linear equations.

METHOD

Three elements of Hypothetical Learning Trajectory (HLT), namely goals, learning task, and hypothetized learning process, were designed by several considerations such as preliminary data (teachers' interview and students' previous learning achievement), national curriculum, and learning observation results. As the national curriculum suggested (Minister of Education and Culture Regulation No. 65, 2013) scientific approach was used as the learning model (5M or observing, asking questions, gathering information, processing information, and communicating).PISA results for Indonesian students have always been an area of concern in which current PISA study reported that the average Indonesian students achievement for mathematics was 379 or 110 marks below the international average of 489 (OECD, 2019). Therefore, PISA-like problems were also embedded in learning materials (worksheets, handbooks, and evaluation tests.

HLT was generated and revised through three learning schemes (Learning I, II, and III). Initial HLT was implemented in Learning I with 12 students. Learning results from Learning I was used as a basis for HLT revision in which the revised HLT was used in Learning II with 35 students. Results from Learning II were used to generate the final HLT, which was then implemented in Learning III with 38 students. Each learning scheme lasted for eight (8) consecutive meetings. Students' learning achievement before and after learning was evaluated with a pretest and posttest.

RESULTS AND DISCUSSION

Hypothetical Learning Trajectory (HLT) for linear equations was designed based on preliminary data, national curriculum, learning observation, and literature study. The designed HLT was then implemented in three learning schemes in which results from previous learning became a basis for revising HLT for the subsequent learning schemes. The final Hypothetical Learning Trajectory (HLT) for the linear equations is presented in Table 1.

As suggested from pretest results in Learning I-III (Table 2), students still have a rather simplistic understanding of linear equations. Students' ability to reason and translate mathematical problems into a mathematical language was still limited, and they tend to use a noncreative method in solving linear equation problems due to their limited knowledge about various techniques for solving the problem. Observation in each learning meeting showed that progression did happen over time if scaffolding was sufficiently used throughout learning activities. Therefore, aside from revision to improve typing errors, the importance of scaffolding also becomes a point of consideration in generating HLT for Learning II and III. The revised HLT was then used in Learning II with 35 students and Learning III with 38 students. Similar to results in Learning I, students in Learning II and III did not have a deep understanding. Errors resulted from the inability to translate mathematical problems into a mathematical language, and a limited understanding of mathematical techniques was also detected. In learning II and III, the high incidence of answering the question by guessing further indicated insufficient understanding.

Table 1. The Final Hypothetical Learning Trajectory (HLT) For Linear Equations

Learning Goals	Learning Activity	Hypothetized Learning Process
Meeting 1: Students can define linear equations of two variables, solve problems related to linear equations of two variables, and create two-variable equations mathematical model for everyday problems. **Learning activities for Meetings I-VIII is in a similar design**: students form groups of five people and each group is asked to solve linear equations problem(s). Learning steps are adjusted to the scientific learning model: observing, asking questions, gathering information, processing information, and communicating.	The problems that students must solve are: 1) Pak Komang wants Rp. 1,000,000.00 to be withdrawn from two ATMs, namely BNI ATM and BCA ATM, each with a different nominal value, namely Rp. 100,000.00 and Rp. 50,000.00. How many cash withdrawal variations can Pak Komang do to take the money? State the probability of withdrawing money at the two ATMs in the form of an equation? In your opinion, what are the similarities between the above problems? 2) Mitha wants to buy chocolate donuts and strawberry donuts at a cake shop. He planned to buy 15 donuts. How many of each donut might Mitha buy? Write down all the possibilities! What is the form of the mathematical equation for the above problem? 3) Which of the following statements will produce two variables linear equations? A rectangular photo frame with a perimeter of 80 cm or a rectangular photo frame that has an area of 375 cm^2?	1) Students can solve problems using a systematic way 2) Students may experience confusion in solving the problems. 3) With the help of literature, students can define a linear equation of two variables. 4) Students can make mathematical models correctly. Possible answers for example a) The amount of money taken at BNI ATMs and the amount of money taken at BCA ATMs is IDR 1,000,000, b) BNI ATM money + BCA ATM money = IDR 1,000,000, c) x + y = 1,000,000 (students already know how to write an equation by assuming the money taken at a BNI ATM as x and the money taken at a BNI ATM as y), d) 100,000x + 50,000y = 1,000,000, etc. 4) Conduct necessary scaffolding efforts (guiding question, prop-letic questioning, revoicing, etc.).
Meeting 2: Students can define a two-variable linear equation system, can create a mathematical model in the form of a two-variable linear equations system for everyday problems, and can distinguish two-variable linear equations from a two-variable linear equation system.	Students have to solve the problems: 1) Wayan wants to buy shirts and pants at a shop. He saw a man paying IDR 175,000.00 for two shirts and three pairs of pants. Then a woman also paid IDR 200,000 to the cashier to purchase three shirts and two pants. Make a mathematical model for the situation! The equation form of the above mathematical model is a Two-Variable Linear Equation System. What is a two-variable system of linear equations? What is the difference between two-variable linear equations and two-variable linear equations system? 2) The difference between the ages of a father and his daughter is 26 years, whereas five years ago, the sum of their ages was 34 years. What is the appropriate mathematical model for this situation? 3) In the parking lot consisting of motorbikes and cars, there are 25 vehicles. The total number of wheels is 80 pieces. If the number of motorcycles is represented by x and the number of cars is expressed by y, what is the two-variable linear equations of the situation?	1) Students can define a two-variable linear equation system, create a mathematical model in the form of a two-variable linear equation system for everyday problems, and distinguish two-variable linear equations from a two-variable linear equation system. 2) Students might make mistakes, for example, x = shirt, y = pants so 5x + 5y = IDR 375,000, -. Students may not be able to distinguish two-variable linear equations from a two-variable linear equation system so they may make mistakes, for example thinking that two-variable linear equations have one mathematical equation, and two-variable linear equation system has two mathematical equations. Conduct necessary scaffolding efforts (guiding question, propletic questioning, revoicing, etc.).

Table 1. (continued)

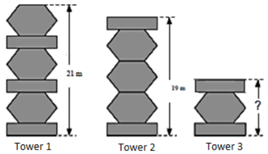
Meeting 3: Students can solve two-variable linear equations using the graphical method	Students have to solve the problems: 1) three towers have different heights and are composed of hexagons and rectangles. Make a mathematical model for the problem and calculate how tall is tower 3? (Hint: use the graphical method)  2) Asep buys 2 kg of mangoes and 1 kg of apples, and he has to pay Rp15,000.00, while Intan buys 1 kg of mango and 2 kg of apples for Rp 18,000.00. What is the cost of 5 kg of mango and 3 kg of apples?	Students can solve problems from the two-variable linear equation system with the graphical method. Students' prior knowledge and with the help of literature in textbooks and other sources, students can find solutions to a three-variable system of linear equations. Conduct necessary scaffolding efforts (guiding question, propletic questioning, revoicing, etc.).
Meeting 4: Students can solve the problems of two-variable linear equations system using the substitution method	Students have to solve: 1) Mr. Nyoman owns a vehicle rental company consisting of sedans and vans with daily rental fees listed on the poster. One day last week, Mr. Nyoman's total income was Rp. 30,000,000.00 for sedans and vans. On that day, his company leased 122 vehicles. How many sedans and vans were rented that day?	Students can solve the two-variable linear equation system with substitution method. Students might solve the problems by thinking that $2x + 3y = 85$. Conduct necessary scaffolding efforts (guiding question, propletic questioning, revoicing, etc.).
Meeting 5: Students can solve two-variable linear equations system using elimination method	Students have to solve: 1) Putu and Made buy snacks and milk of the same type but at different stores. Putu bought three snacks and two glasses of milk at the Indomaret shop for Rp 12,900.00. At the same time, another customer bought two snacks and three glasses of milk of the same type for Rp 16,350.00. With Rp 19.000,00 in cash, Made got four snacks and three glasses of milk at the Alfamart store, then other customers bought three snacks and four glasses of milk for Rp 22,650.00. How much do one snack and one milk cost in each store? 2) Lia's age is seven years older than Irvan's age, while the sum of their ages is 43 years. How old are they?	Students can solve the problem of with the Elimination Method Conduct necessary scaffolding efforts (guiding question, propletic questioning, revoicing, etc.).
Meeting 6: Students can solve two-variable linear equations system with mixed methods (elimination and substitution)	The problems that students have to solve are: 1) A house has a water reservoir. Through a pipe, water flows from the reservoir into the bath. The volume of water in the bath after 5 minutes is 25 liters and after 12 minutes is 46 liters. The volume of water in the bath after t minutes is expressed as liters, where V0	Students can solve the problem using the mixed method Conduct necessary scaffolding efforts (guiding question, propletic questioning, revoicing, etc.).

Table 1. (continued)

	is the volume of water in the bath before the water is drained and a is the water flow rate per minute. What is the volume of water in the bath before the water is drained? How much water is flowing every minute? What is the volume of water in the bath after 27 minutes? 2) During a canoe rowing competition, a participant rowed a canoe for 12 kilometers against the current for 3 hours and 12 kilometers following the current for 2 hours. If the current velocity is considered constant, determine the canoe's velocity while in the water and water current's velocity. Clue: (i) The distance traveled (d) is equal to the speed (r) times the time traveled (t), d = r × t, (ii) when against the river flow (a): average boat velocity – river current velocity = canoe's velocity when against the flow. When going with the river flow (b): average boat velocity + river current velocity = canoe's velocity when going with the river flow".	
Meeting 7: Students can solve fractional two-variable linear equations system	Students must solve: 1) Mrs. Ani and Mrs. Wati shop for necessities to make cakes at a shop. Mrs. Ani bought 1/2 kg of flour A and 3/4 kg of flour B. Meanwhile, Mrs. Wati bought 1.5 kg of flour A and 0.25 kg of flour B. If Mrs. Ani had to pay Rp 13,000.00 for her groceries, and Mrs. Wati pays Rp 15.000,00, determine the price of flour A and flour B! 2) The rectangle's width is 2/3 of its length. The sum between the width and half of the length is 12 cm. Determine the length and width of the flat shape!	Students may write an equation in the form of a fraction or convert a fraction to a decimal, for example 0.5x + 0.6y = 13,000 and 1.5x + 0.25y = 15,000. Students may also solve problems using the graph, elimination, substitution, or mixed methods. Conduct necessary scaffolding efforts (guiding question, propletic questioning, revoicing, etc.).
Meeting 8: Students can solve the nonlinear equations	Sum of squares of two times Kadek's age equals to sum of squares of Putu's age plus 7. Sum of squares of two times Putu's age plus sum of squares of three times Kadek's age equal to 14. How old is Putu and Kadek?	Students can solve the problem of a nonlinear equations. Students may directly solve the problem without doing the example or by assuming x2 = p and y2 = q. Students may also solve problems using graphs, elimination, substitution, or mixed methods. Conduct necessary scaffolding efforts (guiding question, propletic questioning, revoicing, etc.).

Table 2. Students' Linear Equations Pretest and Posttest Result

Learning Scheme	N	Pretest		Posttest	
		Score Range	Avg. ±SD	Score Range	Avg. ±SD
Learning I	12	35-60	51.25 ± 7.7	60-80	72.91 ± 6.2
Learning II	35	40-65	53.14 ± 7.9	65-85	74.00 ± 6.5
Learning III	38	40-65	53.28 ± 7.1	65-90	74.34 ± 7.2

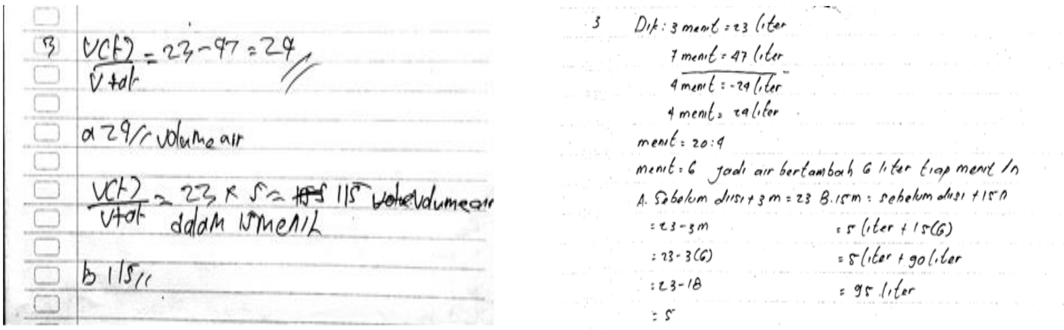


Figure 1. Student Answer Examples for Question No. 3 in the pretest: "*Calculate water bath volume after 15 minutes if water volume in a bath is 23L after 3 minutes and 47L after 7 minutes with V0 is water volume before water flowed to the water bath and a is water debit/minute.*"

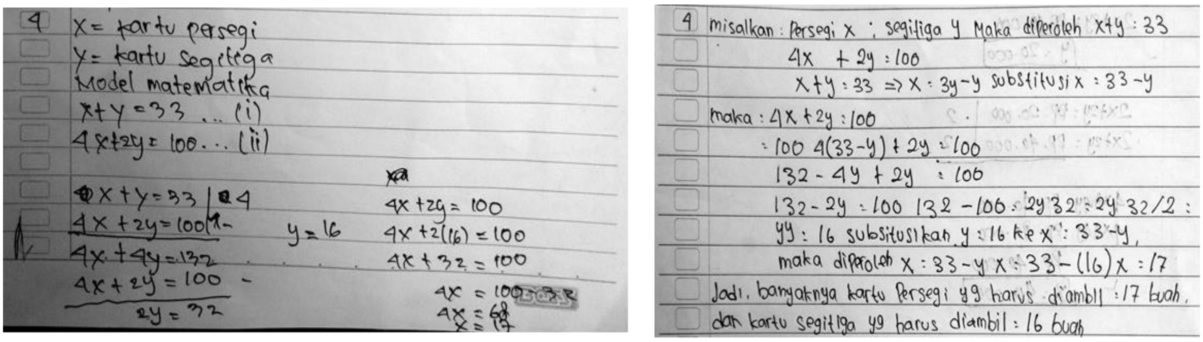


Figure 2. Student Answer Examples for Question No. 4 in the posttest: "*Mr. Purwoko's son Angga has a deck of cards. The whole card can be divided into two parts according to its shape. One type is square, in which there is a picture of a buffalo and four birds. The other is a triangle with a picture of a buffalo and two birds. How many square and triangular cards must be drawn from the deck so that the total number of images of buffalo is 33 and the number of images of birds is 100.*"

Studies have reported obstacles that students found when learning linear equations (for example Huntley et al., 2007; Birinci et al., 2014; Adu et al., 2015). Kusmaryono (2018) stated that if students' answers reflected a lack of knowledge about concepts or symbols and a distorted understanding of the specific principle, rule, theorem, or definition, the students could be categorized as a student with insufficient mathematical comprehension. As shown in their answers (Figure 1), students in this study were: 1) having a rather shoal understanding of linear equation concepts, definitions, symbols, or formula, 2) unable to translate mathematical problems into a mathematical language, and 3) having a limited repertoire of mathematical techniques, which reflected their insufficient comprehension of linear equations. This result is similar to previous studies (Huntley et al., 2007; Hewitt, 2012), which found that students have a superficial understanding of linear equations (Huntley et al., 2007) and frequently making processing errors when dealing with linear equations questions (Hewitt, 2012). High incidence of answering the question by trial and error (guessing the answers) was also found in this study, in which Beal, Qu, and Lee (2008) study found that students who did not feel confident in their math ability and feels that math was difficult were more likely to guess. Further, Caglayan and Olive (2010) found that disconnection between mental operations and physical operations is the leading source for students obstacles and difficulties in learning linear equations.

Improvement in learning can be achieved through scaffolding (Bakker, Smit, and Wegerif, 2015) in which features of scaffolding such as contingency based on students' needs, gradual support withdrawal over time, and responsibility transfer from teachers to students (van de Pol, Volman, and Beishuzen, 2010) facilitate mathematics learning improvement (Moschkovich, 2015). In designing scaffolding, studies suggested that scaffolding will be most effective if teachers can scaffold pupils learning by employing a range of teaching approaches (Anghileri, 2006; Moschkovich, 2015).

In one weekly class meeting, students were asked to solve: "During a canoe rowing competition, a participant rowed a canoe for 12 kilometers against the current for 3 hours and 12 kilometers following the current for 2 hours. If the current velocity is considered constant, determine the canoe's and water current's velocity. Clue: (i) The distance traveled (d) is equal to the velocity (r) times the time traveled (t), d = r × t, (ii) when rowing against the river flow (a): average boat velocity – river current velocity = canoe's velocity when against the flow. When going with the river flow (b): average boat velocity + river current velocity = canoe's velocity when going with the river flow". In assisting the students to answer the question, scaffolding was conducted such as by giving guiding questions (written in bold and italic) as depicted in the following conversation between researcher (R) and one of the students (S).

R : Did you find any problems?
S : Yes. How can we find the mathematical model?
R : Have you read the clue in the question?
S : I have, Mam.
R : ***If so, what do you think can be x and y variable?***
S : Average boat velocity and river current velocity
R : That's right!
S : But the information in the question were only distance and time…
R : Well, the clue stated that velocity = distance/times. You can use this information.
S : Ok, Mam. We will try it.

In another meeting, when the students were given a task to formulate a mathematical model for calculating total wheels in a parking lot, scaffolding was delivered by proleptic questioning and revoicing. In the task, students were given information that there are 25 vehicles (motorbikes and cars) in the parking lot, with 80 as a total number of the wheel in that parking lot.

S : Mam, what do you think about our model?
R : ***Do you think it is right?***
S : Yes, Mam.
R : ***How can you arrive at your proposed model?***
S : Because the parking lot consisted of 25 vehicles, so x + y = 25. The total number of wheels in the parking lot is 80. Motorbike has two wheels whereas car has four. Therefore, 2x + 4y = 80.
R : ***So, the basis of the model is how many wheels that motorbike and car have?***
S : Yes.
R : Ok, it is a good idea.

After learning, posttest from Learning I-III showed that some students still struggle with more complex problems in which they failed to arrive at a correct solution. However, most students' answers reflected a more structured understanding. Learning progression can be seen in students' answers in pretest and posttest (Figure 1 and 2). In the pretest, students were asked to calculate water bath volume after 15 minutes if water volume in a bath is 23L after 3 minutes and 47L after 7 minutes with V_0 is water volume before water flowed to the water bath and a is water debit per minute. As depicted in Figure 1, students could not answer the question correctly and were even confused about what they have to do with the available information. For a similarly complex question in the posttest, students' answers reflected a more structured answer (Figure 2).

Sztajn, Confrey, Wilson, and Edgington (2012) summarize that learning trajectory became a unifying element for instructions when teachers organize teaching from a learning trajectory perspective. In this current study, designing Hypothetical Learning Theory for teaching linear equations in which learning goals (PISA and national curriculum consideration) embodied in teaching activities and presumed obstacles in learning becomes a sound and effective basis for preparing assistance efforts blended within linear equations learning instruction.

CONCLUSION

Hypothetical Learning Theory (HLT) for teaching linear equations serves as a unifying element for learning linear equations. HLT implementation in three learning schemes suggested the importance of scaffolding in learning linear equations. Sufficient and strategic scaffolding can improve student's understanding and facilitate the students in overcoming obstacles when learning linear equations.

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