



Journal of Didactic Studies

Published by: Indonesian DDR Development Centre
Directorate of Innovation and University Center of Excellence, Universitas Pendidikan Indonesia
Journal homepage: <https://ejournal.upi.edu/index.php/jds>

jds
didactic

Prospective Analysis on the Topic of Similarity: Designing a Hypothetical Learning Trajectory to Overcome Learning Obstacles

K. S. Kuncoro^{1*}, D. Juandi¹, R. Hidayat², and A. Prabowo³

¹Universitas Pendidikan Indonesia, Bandung, Indonesia

²Universiti Putra Malaysia, Selangor, Malaysia

³Universitas Negeri Semarang, Semarang, Indonesia

Correspondence: E-mail: kridakuncoro@upi.edu

ABSTRACTS

This research is part of Didactical Design Research (DDR) aimed at developing a Hypothetical Learning Trajectory (HLT) in the concept of similarity learning. This study focuses on the prospective analysis stage within DDR. The HLT developed in this research is the result of identifying potential learning obstacles that may arise during the process of similarity learning. Data was collected through five steps in the prospective analysis: 1) cognitive development of seventh-grade students in middle school; 2) research on the topic of similarity and related research findings; 3) curriculum analysis of mathematics textbooks using praxeological theory; 4) reflection and evaluation of learning obstacles; 5) anticipation of student responses and pedagogical strategies. Data collection techniques used were literature study and Praxeology analysis within the Anthropological Theory of Didactics (ATD). The obtained data was then analyzed thematically to identify and understand learning obstacles in the topic of similarity, as well as to design an HLT to address these obstacles. This research revealed difficulties experienced by students in understanding the concept of similarity, such as a lack of initial understanding of scale and magnification concepts, difficulties in comprehending the relationship between ratios and scale factors previously learned, and a lack of relevant task design sequence related to the application of

ARTICLE INFO

Article History:

Received 29 Jan 2023

Revised 30 Apr 2023

Accepted 25 May 2023

Available online 11 June 2023

Keyword:

Concept of similarity,
Didactical design research,
Hypothetical learning trajectory,
Learning obstacles,
Online teaching materials,
Praxeology.

similarity in everyday life. The conclusion of this research is that designing an effective online HLT to overcome learning obstacles in the topic of similarity requires a student-centered approach, the use of active learning strategies, and interactive student engagement. Additionally, it is important to encourage students to think critically and connect concepts to everyday life.

© 2023 Universitas Pendidikan Indonesia

1. INTRODUCTION

The results of the Program for International Student Assessment (PISA) in Indonesia reveal that a significant number of 15-year-old students lack proficiency in reading comprehension and basic mathematical concepts (OECD, 2019). Unfortunately, these scores have shown little improvement over the past decade, indicating disparities in learning quality across regions and socio-economic groups. The COVID-19 pandemic has further exacerbated these challenges.

To overcome the learning setbacks caused by the pandemic, the Merdeka Curriculum was developed in Indonesia. This curriculum emphasizes flexibility, essential content, and the development of students' character and competencies (Hadi et al., 2023). Project-based learning is a key component of the Merdeka Curriculum, focusing on nurturing soft skills and character traits in line with the Pancasila student profile (Fitriyani et al., 2023). Additionally, the curriculum prioritizes essential content to allow for in-depth learning in vital competencies such as literacy and numeracy (Reid & Reid, 2017). The Merdeka Curriculum also empowers teachers to implement differentiated learning based on students' abilities and adapt to local contexts and content.

Understanding the concept of similarity in mathematics plays a crucial role in developing students' problem-solving abilities and logical thinking (Seago et al., 2013). This concept enables students to recognize patterns and proportional relationships among objects or geometric figures. Therefore, effective teaching of similarity becomes highly important in the learning process (Simsek & Clark-Wilson, 2018). Referring to the principles of the Merdeka Curriculum, teaching the concept of similarity can be tailored to the individual needs of students and adapted to local contexts and content.

To achieve effective teaching, it is crucial for teachers to have a strong scientific knowledge of the concept of similarity (Seago et al., 2013). Teachers need to have a deep understanding of the definition of similarity, its inherent properties, and the ability to identify and apply this concept in various contexts (Simsek & Clark-Wilson, 2018). With a solid understanding, teachers can provide clear explanations and facilitate a deep understanding among students. Additionally, the textbooks used in the teaching process also play a significant role (Clivaz & Miyakawa, 2020). Evaluating the facts presented in textbooks can help assess the sustainability of explanations and examples given in teaching similarity. It is important to ensure that textbooks provide adequate explanations of the concept of similarity, present relevant examples, and foster a good understanding among students.

In the context of education in Indonesia, the concept of the Merdeka Curriculum provides opportunities for teachers to develop the teaching of similarity in a more flexible manner (Hadi et al., 2023). The Merdeka Curriculum grants freedom and creativity to schools and teachers in designing lessons that align with the needs and characteristics of students.

However, it is important to acknowledge that there are currently limitations in the available textbooks for students. These issues need to be further explored in the context of teaching similarity. Existing textbooks may still not present the material of similarity comprehensively or provide sufficient relevant examples for students. Therefore, a comprehensive review of the textbooks used in teaching similarity is necessary to find appropriate solutions for enhancing the quality of learning.

Technology integration also plays a pivotal role in differentiated learning, as advancements in science and technology impact various aspects of society (Szymkowiak et al., 2021). To facilitate effective learning, it is crucial to integrate multimedia resources that provide access to real-world representations and offer multiple perspectives (Auh & Sim, 2019). While learners now have access to a wide range of educational materials, traditional books remain essential in education (Ko & Rossen, 2017). Integrating multimedia resources effectively with technology can enhance the teaching and learning process, making it more efficient and engaging (Serrano et al., 2019).

One emerging technology that has gained traction in the education sector is the use of Online Teaching Materials. These materials leverage digital resources to create interactive and engaging learning experiences for students. Online Teaching Materials offer flexibility, interactivity, and contextual learning opportunities, enabling educators to support student capabilities effectively (Aljawarneh, 2020).

Online Teaching Materials present opportunities for educators to create and enhance effective and innovative learning experiences in mathematics education, facilitating students' understanding of geometric concepts. By leveraging these materials, teachers can improve problem-solving skills and foster deeper learning experiences. Understanding students' cognitive development stages, influenced by both individual and environmental factors (Bjorklund, 2022; Hutchison, 2018), is crucial in designing online teaching materials tailored to their needs.

The purpose of this study is to investigate the construction of a Hypothetical Learning Trajectory (HLT) for the instruction of the similarity topic, utilizing the principles of praxeological theory. The primary goal is to identify and elucidate essential elements including student characteristics, learning outcomes, and objectives related to the subject of similarity. The outcomes of this research will guide the creation of online instructional materials that foster problem-solving abilities, align with the curriculum, and cater to the specific needs of the students.

2. METHODS

This qualitative research is a preliminary study of Didactical Design Research (DDR) (Suryadi, 2019b), which consists of three main stages: prospective analysis, metapedadidactic analysis (didactic-pedagogical situation), and retrospective analysis. In the DDR research stage, this study is currently in the prospective analysis phase, where the didactic situation is evaluated before the learning process begins (Suryadi, 2019a).

The research subjects in the characteristic analysis stage were seventh-grade students who were studying similarity material. The data collection technique used was a literature study. The literature study aimed to gather relevant research results on topic of similarity. Additionally, a review of core competencies, essential competencies, concepts, and assignment designs was conducted using reference textbooks. Praxeology was employed to identify the problems that arise in the task design of similarity material (Bosch et al., 2017). The book being analyzed is a class VII math book published by the Indonesian Ministry of Education (Susanto et al., 2022).

The HLT is developed based on the analysis of learning obstacles, students' thinking stages, and an examination of the curriculum that aligns with the concepts of similarity that students need to master. The HLT is structured around the learning objectives to be achieved, supporting activities, and mathematical hypotheses that arise in students and correspond to their thinking levels (Bakker & Smit, 2018). The stages undertaken by the researcher in developing the HLT are presented in Figure 1.

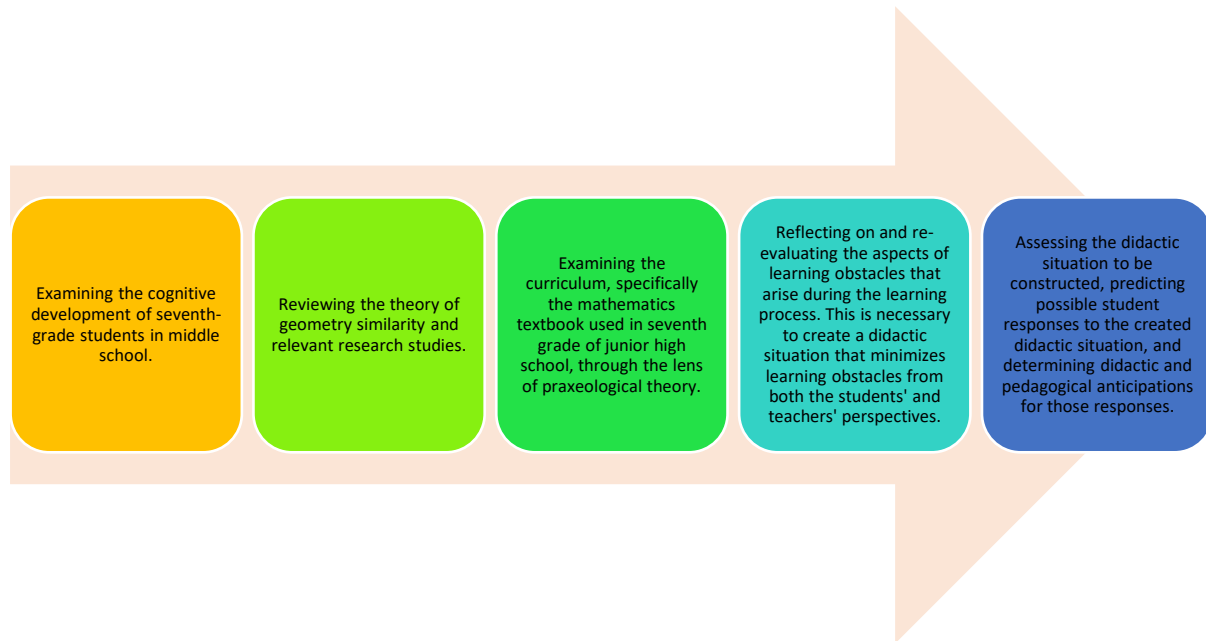


Figure 1. Steps in Developing HLT in the Prospective DDR Phase

3. RESULTS AND DISCUSSION

Results

The results of the prospective DDR phase in this study are 1) Cognitive development of seventh-grade students in middle school; 2) Investigation on the topic of similarity and relevant research findings; 3) Curriculum analysis of math textbook with praxeological theory; 4) Reflection and evaluation of learning obstacles; 5) Anticipation of student responses and pedagogical strategies.

Cognitive development of seventh-grade students in middle school

In the study of the cognitive development of seventh-grade students in middle school, several theories of cognitive development can provide relevant guidance. One of the relevant theories is Jean Piaget's theory of cognitive development. According to Piaget, cognitive development occurs through a series of stages, and in the formal operational stage, seventh-grade students begin to develop abstract and logical thinking abilities (Kilag et al., 2022). This stage is important in learning complex concepts such as similarity (Kazi & Galanaki, 2019).

Additionally, the sociocultural theory by Lev Vygotsky can also support learning on the topic of similarity. According to Vygotsky, learning occurs through social interaction and cultural influence (Mcleod, 2022). In the context of the topic of similarity, collaborative learning activities and discussions can enhance students' understanding and application of the concept.

The meaningful learning theory by David Ausubel also has relevance in learning about similarity. Ausubel emphasizes the importance of prior knowledge in learning new concepts

(Agra et al., 2019). In learning about similarity, teachers can activate students' prior knowledge of related concepts and relationships to facilitate meaningful learning.

Furthermore, Jerome Bruner's constructivist theory emphasizes the construction of knowledge by learners. According to Bruner, learners create mental representations of concepts through exploration and problem-solving (Myers, 2021). In the context of learning about similarity, students can engage in hands-on activities, use visual aids and manipulatives, and solve problems to build their understanding of the concept.

The multisensory mathematics theory by Zoltan Dienes can also be used in learning about similarity. This theory emphasizes the use of multisensory experiences to enhance mathematical learning (Fägerstam, 2014). In learning about similarity, the use of visual aids, manipulatives, and interactive technology can engage students' multiple senses and facilitate a deeper understanding of the concept.

Investigation on the topic of similarity and relevant research findings

The investigation of the topic of similarity and relevant research findings also has implications for the development of the Hypothetical Learning Trajectory (HLT) in teaching this concept. The HLT serves as a guide for educators in planning a coherent sequence of learning experiences that support students' progression in understanding the topic of similarity (Duschl et al., 2011). By examining the research findings on the topic of similarity, educators can identify key learning obstacles, misconceptions, and challenges that students commonly face during the learning process (Qian & Lehman, 2017). This information can inform the design of the HLT by incorporating specific instructional strategies and interventions that address these challenges and promote deep understanding (Bakker & Smit, 2018).

Additionally, the research findings can help in determining appropriate sequencing of learning activities and tasks within the HLT to scaffold students' learning and promote gradual development of their understanding of the topic of similarity. Research may suggest starting with concrete examples and gradually transitioning to abstract representations or providing opportunities for students to engage in problem-solving tasks that require the application of similarity concepts.

Furthermore, the integration of technology tools and resources within the HLT can be informed by research on the effectiveness of technology in supporting students' learning of the topic of similarity (Simsek et al., 2022). This may involve incorporating interactive software, virtual manipulatives, or online simulations that provide opportunities for students to explore and manipulate concepts related to similarity (Simsek & Clark-Wilson, 2018).

By incorporating the insights and findings from research into the development of the HLT, educators can create a more effective and targeted instructional trajectory that supports students' cognitive development in the topic of similarity and enhances their learning outcomes (Cox, 2013). The HLT can serve as a framework for educators to plan and implement engaging and meaningful learning experiences that promote students' conceptual understanding and application of similarity concepts in real-world contexts.

Curriculum analysis of math textbook with praxeological theory

The analysis of the seventh-grade mathematics curriculum textbook using the praxeological theory from ATD involves evaluating and examining the curriculum used in teaching the topic of similarity. It is important to investigate the mathematics textbook used in the seventh grade and identify how the concept of topic similarity is integrated into the learning materials. This analysis requires a deep understanding of the curriculum structure

outlined in the textbook's task design, the sequence of lessons, and the emphasis placed on key aspects of topic similarity.

Several previous studies have analyzed how tasks contained in textbooks are used in mathematics learning activities (Shinno & Mizoguchi, 2021; Takeuchi & Shinno, 2020; Wijayanti & Aufa, 2020). In this section, the researcher investigates the opportunities and potential offered by mathematics textbooks in constructing knowledge and the interrelationships of pieces of knowledge that will be taught in the topic of similarity. Similarity is often conceptualized in discrete terms as a numerical relationship between two numbers, especially at the secondary school level (Seago et al., 2013). Similarity is also defined as having the same shape but not necessarily the same size. Other conceptualizations of congruence are based more on relationships. Understanding the concept of congruence underlies the understanding of several other concepts in geometry.

Math assignments often provide multiple solution approaches and strategies. These strategies are often not seen in the final solution of a task. In addition, with sufficient time, it is possible for students to use strategies and approaches that are sufficient, but not optimal. To analyze the design task, the researcher used mathematics textbooks that are used in schools in general. Textbooks have a role in learning mathematics for students, while for teachers textbooks are used to plan and teach mathematics in schools (O'Halloran et al., 2018). Even the instructions in the classroom are usually arranged and conveyed through textbooks. Continuity of knowledge contained in the textbook is very important for learning (Amaral-Schio & Hollebrands, 2021). This continuity principle is reflected in the design task arrangement. Relationships between tasks can provide opportunities for students to learn, or furthermore, how to organize the knowledge learned.

ATD presents a conceptual framework that outlines the understanding of mathematical knowledge in human actions using praxeology (Chevallard & Bosch, 2020). Table 1 shows the topic of similarity and the learning objectives to be achieved.

Table 1. Main topics that focus on similarity in textbooks

Chapter (Grade, objective, page number)	Objectives
Chapter 5 Similarity (7 th grade, page 159)	<ol style="list-style-type: none"> 1. Determining the relationship between angles on intersecting lines and on two parallel lines intersected by a transversal. 2. Estimating the measure of an angle. 3. Utilizing information about angles (complementary, supplementary, vertical, and corresponding angles) in plane figures to solve problems involving unknown angles. 4. Using the congruence criteria to determine if two triangles are similar. 5. Applying the congruence criteria to solve problems.

The theoretical basis for conducting the analysis in this study is based on an analysis of praxeological theory which aims to analyze task design in mathematics textbooks (Chevallard & Bosch, 2020). Praxeology in ATD theory consists of two blocks: praxis and logos. Praxeology implies that the mathematical content of a textbook is analyzed in terms of praxis and logos blocks. Each block has two components: type of task (T) and technique (τ) for the praxis block ('know-how'), and technology (θ) and theory (Θ) for the logos block ('knowledge'). The type of task shows the problem of the task given, while the technique is the way of doing the task. Technology is a form of explaining and justifying technique, whereas theory is used to explain or justify technology (Shinno & Mizoguchi, 2021). This praxeological theory provides an

understanding of the arrangement of material in a textbook from a different point of view. To identify each type of task, the researcher uses four τ which are based on the epistemological theory of knowledge (Audi, 2011) namely perceptual (τ_1), memorial (τ_2), introspective (τ_3) and a priori (τ_4).

The perceptual concept (τ_1) in the context of geometry refers to an individual's ability to acquire knowledge about the shapes, sizes, and positions of geometric objects through visual observation or other senses. Through perception, students can recognize geometric similarities between objects or flat shapes in their environment. The memorial concept (τ_2) in geometry relates to students' ability to store and retrieve information about previously learned geometric similarities. Through memory, students can recall facts, formulas, or principles related to the similarity between objects or congruent triangles. The introspective concept (τ_3) in geometry is related to students' ability to reflect on and observe their understanding, thoughts, feelings, or personal experiences regarding geometric similarity. Through introspection, students can gain a deeper understanding of geometric concepts and relationships between angles, as well as identify strengths and weaknesses in their own understanding. The a priori concept (τ_4) in geometry reflects knowledge acquired deductively or based on logical reasoning without relying on empirical experience. In the context of geometry, a priori knowledge can refer to understanding the properties and basic principles of geometry, such as angle properties, triangle similarity, or line equations, which can be obtained through rational thinking and mathematical logic. The T and τ in the similarity material from mathematics textbooks in Indonesia are presented in Table 2.

Table 2. Types and number of tasks on the concept of Similarity

Task	Description	τ
T1	Exploring intersection angles	perceptual (τ_1), memorial (τ_2).
T2	Determining relationships between angles	perceptual (τ_1), memorial (τ_2).
T3	Angle relationships within a triangle	perceptual (τ_1), memorial (τ_2), introspective (τ_3).
T4	The meaning of similarity in image enlargement	perceptual (τ_1), memorial (τ_2), introspective.
T5	Similarity in triangles through proportional scaling	perceptual (τ_1), memorial (τ_2), introspective (τ_3).

Logos block consists of two parts, namely technology (θ) and theory (Θ). Logos Blok examines the justification of implementation (technology (θ)) in the design task (T) and the type of technique (τ) used based on the underlying theory (Θ). The relation between praxis and block logos is presented in Table 3.

Table 3. The relationship between praxis and logos block in praxeology of similarity material

Topic	Type of Task	Technique	Technology	Theory
Similarity	T1: Exploring intersection angles	τ_1, τ_2	θ_1 : Comparing angle sizes by exploring the relationship between angles.	Θ_1 : transformation
	T2: Determining relationships between angles	τ_1, τ_2		
	T3: Angle relationships within a triangle	τ_1, τ_2, τ_3		
	T4: The meaning of similarity in image enlargement	τ_1, τ_2, τ_3	θ_2 : Comparing scales (enlarged and reduced) of both figures.	
	T5: Similarity in triangles through proportional scaling	τ_1, τ_2, τ_3		

Reflection and evaluation of learning obstacles

In the development of the Hypothetical Learning Trajectory (HLT). Through this reflection, common learning obstacles can be identified, such as a lack of initial understanding of scale and magnification concepts, difficulties in comprehending the relationship between ratios and scale factors previously learned, and a lack of relevant task design sequence related to the application of similarity in everyday life.

Based on Table 2, it is crucial to include a priori knowledge (τ_4) in teaching the concept of similarity. Currently, in the arrangement of tasks in the similarity topic found in mathematics textbooks, a priori knowledge (τ_4) is not adequately covered. A priori knowledge refers to knowledge that is independent of experience and is based on reasoning or deduction. In the context of the concept of similarity, a priori knowledge plays a significant role in understanding and formulating the concepts of similarity. It involves logical deduction, axioms, definitions, and principles that form the basis of reasoning about similarity. By including a priori knowledge in teaching the concept of similarity, students can develop a deeper understanding of the concepts and their relationships. This enables them to make logical connections between the concepts of similarity, draw conclusions, and apply the principles of similarity in problem-solving situations. Without a priori knowledge, students may struggle to grasp the fundamental principles of similarity and rely only on perceptual or memorized knowledge, which can limit their ability to think critically and effectively apply the concepts of similarity. It is essential to recognize the importance of a priori knowledge (τ_4) in teaching the concept of similarity and ensure its inclusion in task design and learning materials. Providing students with a solid foundation of a priori knowledge empowers them to engage in higher-order thinking (Ghanizadeh et al., 2020), make connections between different concepts of similarity, and apply their knowledge to solve problems involving similarity more effectively.

Table 1 also supports this statement, one task design that is a priori in nature is comparing the properties of similarity with other concepts. For example, asking students to compare the properties of similarity with other geometric concepts, such as congruence. Asking students to explain the differences and similarities between the concepts of similarity and congruence in terms of their properties. However, the arrangement of materials in mathematics textbooks does not facilitate this, as the similarity topic is not connected at all to the concept of congruence.

By gaining a better understanding of these obstacles, the HLT can be designed to effectively address them (Bakker & Smit, 2018). For instance, the HLT can incorporate sequential learning steps, starting from concept introduction to guided practice, to help students better grasp the properties of congruence and similarity. Additionally, the HLT can include activities that encourage concept exploration through manipulating geometric objects and applying them in real-world contexts.

In the development of the HLT, the results of reflection and evaluation can also assist in determining the most effective learning sequence (Jupri et al., 2019). By understanding common mistakes that occur, the HLT can guide students in gradually overcoming these obstacles by providing appropriate support and additional exercises based on their understanding level (Utami et al., 2022). Furthermore, the results of reflection and evaluation can provide insights into suitable teaching approaches, effective evaluation methods, and more responsive learning strategies.

Anticipation of student responses and pedagogical strategies.

By anticipating how students might respond to the learning activities and content, educators can design pedagogical strategies that effectively address their needs and promote meaningful learning experiences (Long et al., 2017). In the context of similarity, anticipating student responses involves understanding common difficulties and misconceptions that students may encounter when learning this topic. This research studies have highlighted misconceptions such as a lack of initial understanding of scale and magnification concepts, difficulties in comprehending the relationship between ratios and scale factors previously learned, and a lack of relevant task design sequence related to the application of similarity in everyday life. By being aware of these potential challenges, educators can tailor the HLT to address these specific areas of difficulty and provide targeted support to students (Andrews-Larson et al., 2017).

In designing the Hypothetical Learning Trajectory (HLT) for the topic of similarity, educators have taken into consideration the anticipated responses and needs of students. The HLT consists of four stages that aim to facilitate students' understanding of the topic of similarity in a meaningful and comprehensive way.

The first stage focuses on developing a solid foundation in the basic concept of similarity. Students are introduced to the fundamental principles of geometric similarity and engage in discussions regarding its definition and properties. Through the use of examples and practical exercises, students are given the opportunity to apply the concept of similarity to various geometric shapes, fostering their ability to recognize and comprehend this fundamental concept.

Moving on to the second stage, the HLT delves into the concept of scale transformations. Students are taught about the process of scaling, including concepts such as enlargement and reduction. Additionally, the relationship between scale factors and the properties of similarity is explored and explained. Practical applications of scale transformations on different geometric shapes are introduced, allowing students to practice and solidify their understanding of this transformative process.

The third stage focuses on the application of similarity in problem-solving contexts. Students learn how to apply the concept of similarity to solve various geometric problems. They are presented with real-world scenarios that require the utilization of similarity principles. Through guidance and support, students develop the skills to effectively solve problems using the principles of similarity and enhance their problem-solving abilities in a geometric context.

In the final stage of the HLT, the application of similarity and tessellation in real-world contexts is explored. Students are encouraged to connect the concepts of similarity and tessellation to practical situations in their daily lives. They learn how similarity can be applied in fields such as architecture, design, and model-making. Furthermore, the concept of tessellation is introduced, emphasizing how it can be used to create symmetrical patterns and designs. Students are provided with opportunities to design and create models based on real-life cases, incorporating the principles of both similarity and tessellation. The visual representation of this HLT is presented in Figure 2.

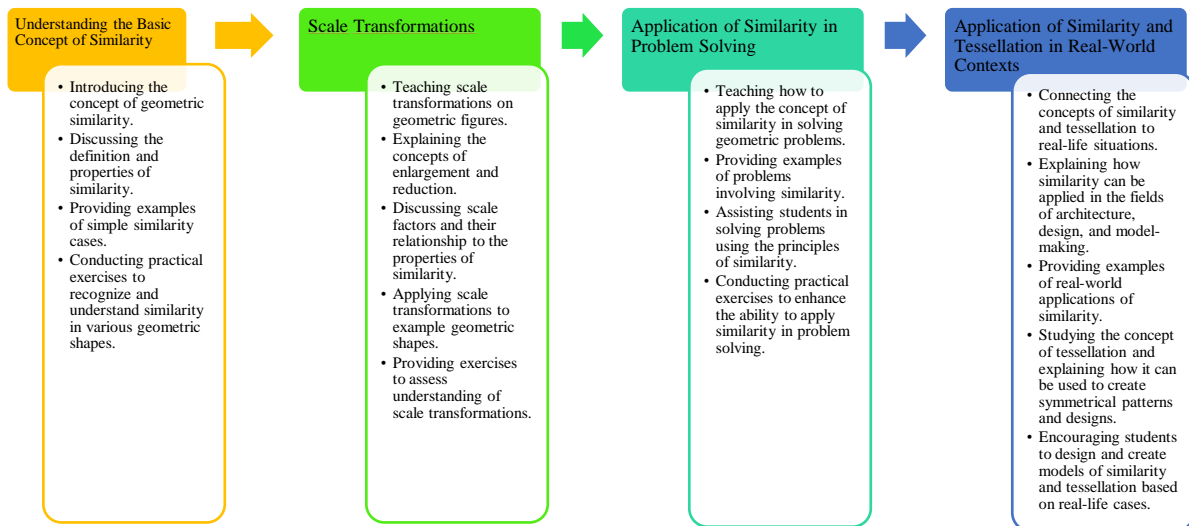


Figure 2. Draft HLT for concept of similarity.

Throughout the HLT, pedagogical strategies are employed to enhance students' understanding of the topic of similarity. These strategies may include the use of visual aids, hands-on activities, and interactive technologies to create an engaging and interactive learning environment. Additionally, the integration of online learning materials plays a crucial role in facilitating the learning process (Kebritchi et al., 2017).

Online learning materials provide a flexible and accessible platform for students to engage with the concept of similarity in geometry. Through online platforms, students can access a wide range of resources such as video lessons, interactive modules (Castro, 2019), and digital tools specifically designed to support their understanding of similarity. These materials offer the advantage of anytime, anywhere access, allowing students to review and reinforce their knowledge at their own pace and convenience. One of the benefits of online learning materials is the opportunity for interactive and immersive learning experiences (Dengel & Mägdefrau, 2018). Through simulations, virtual manipulatives, and interactive visualizations, students can actively explore and experiment with the concept of similarity. They can manipulate geometric figures, observe transformations, and observe the effects of scaling, further deepening their understanding of similarity through interactive engagement.

Online learning materials also provide immediate feedback to students (Martin & Bolliger, 2018). Through online quizzes, assessments, and interactive exercises, students can receive instant feedback on their understanding and progress (Zainuddin et al., 2020). This timely feedback helps students identify areas of improvement, correct misconceptions, and reinforce their understanding of similarity concepts. Furthermore, online learning materials foster collaboration and communication among students (Martin & Bolliger, 2018). Through online discussion forums, collaborative projects, and virtual group activities, students can engage in meaningful discussions, share ideas, and learn from their peers. This collaborative aspect of online learning promotes critical thinking, problem-solving skills, and a deeper understanding of similarity as students exchange perspectives and work together to solve geometry problems.

Discussion

The current curriculum in Indonesia encourages the use of technology in the classroom. Technology has become an integral part of daily life and is crucial in developing the skills needed by children to compete in an ever-changing world (Carstens et al., 2021; Lestari et al., 2022; Parisa et al., 2023; Wulandari et al., 2022). The use of technology in learning can help students learn more effectively and enjoyably, as well as provide access to a wider range of learning resources (Puspita et al., 2022; Rapanta et al., 2020).

Online-based learning materials are technologies that enable students to learn through digital platforms (Haleem et al., 2022). Online-based learning materials can help students learn in a more interactive and engaging manner, as well as provide access to a wider range of learning resources (Prabowo & Juandi, 2020; Rusydiyah et al., 2020). Students can use online-based learning materials to visualize difficult concepts and make them easier to understand (Khan et al., 2017; Kuncoro et al., 2022). Online-based learning materials can also provide a responsive and interactive learning environment that addresses various types of content (Tang et al., 2020).

Furthermore, integrating learning materials with online-based approaches will create a better learning environment. This allows students to have a better learning experience both in the classroom and at home (Sun et al., 2017). Online-based learning materials combine printed books (or their digital formats) with 3D models and 2D graphics. By using online-based technology as a framework for presenting and interacting with audio-visual content (Dakhi et al., 2020), interactive online-based learning materials become the main facility in improving understanding by enabling students to acquire knowledge more quickly (Rahiem, 2020). Technologically, the learning environment using online-based learning materials consists of smartphones (to generate individual user displays) and physical books (Krishnan et al., 2020).

According to Jean Piaget's theory of cognitive development, during the formal operational stage (ages 11 and up), students are capable of logical and deductive thinking and can use abstract concepts in problem-solving (Decano, 2017). The use of online-based learning materials can assist students at this stage by enriching their learning experiences through direct interaction with the real world enhanced by digital additions (Ferri et al., 2020). Online-based learning materials can also help students visualize abstract concepts and understand how these concepts are interconnected, which aids them in problem-solving (Wu & Cheng, 2019).

Research shows that this innovative technology approach allows students to have rich sensory stimulation experiences, enjoyable interactions with content (e.g., reading, listening, and displaying still and moving images in 3D virtual models through online-based learning), and thus enhances the learning process (Aaron & Lipton, 2018; Gillett-Swan, 2017). Additionally, online-based learning materials can support both traditional pedagogical practices (such as reading books) and innovative pedagogical approaches by providing rich media. The implementation of interactive online-based learning materials is a new form of learning that is natural and fully under the control of users (both students and educators). The new interactive approach in connecting traditional pedagogical approaches (such as reading printed books), common device capabilities (such as handheld devices with cameras), and the potential of multimedia technology (audiovisual interpretation technology) can lead to better understanding, better knowledge acquisition, and better learning experiences.

However, based on the praxiological theory of ATD construction (Chevallard & Bosch, 2020), some adjustments need to be made in delivering the Similarity material. Table 2 shows that the presentation stages of similarity material start with exploring intersection angles (T1). In T1, the main emphasis is on intersecting angles rather than the concept of similarity. This

can cause students to focus on aspects that are not directly related to similarity, such as the relationship between intersecting angles and lines. This shortcoming leads to students not gaining a comprehensive understanding of the concept of similarity, including the relationship between the sizes of corresponding sides and angles. As a result, students may struggle to apply the concept of similarity in broader contexts, such as real-life problem-solving or comparing scale ratios. In terms of knowledge construction, the results of the praxeology analysis on the concept of similarity show that the student learning process does not allow students to arrive at knowledge as well as justification for similarity material. This is indicated by the T arrangement where τ does not contain τ_4 (a priori). This lack of knowledge construction can cause students to experience epistemological obstacles. Therefore, before developing online-based learning materials, adjustments need to be made to the task design sequence for the concept of similarity.

The relationship between the knowledge nodes traversed during learning and the information received by students is crucial in helping students build and integrate new knowledge into their existing knowledge structure (Taranto et al., 2020). When students learn something new, this information can be related to their pre-existing knowledge, making it easier for students to understand new concepts. According to Ausubel's theory of learning, there are two types of knowledge nodes: independent knowledge nodes and dependent knowledge nodes (Agra et al., 2019; Wang et al., 2020). Independent knowledge nodes are concepts or ideas that can be related to many other concepts and have their own value. Dependent knowledge nodes are concepts or ideas that do not have meaning on their own but only have meaning in relation to other concepts. According to this theory, students will find it easier to understand new concepts if these concepts can be connected to independent knowledge nodes they already possess, rather than dependent knowledge nodes (Barnett et al., 2013). Therefore, it is important to help students find connections between their existing knowledge nodes and the new concepts they are learning, making it easier for them to understand and integrate these new concepts into their existing knowledge structure (Barnett et al., 2013).

In the case of similarity materials, an important independent knowledge node is the concept of similarity itself. This knowledge node has its own value and can be related to many other concepts, such as the concepts of comparison, proportion, and other geometric concepts. Dependent knowledge nodes in similarity materials include the concepts of equilateral triangles, isosceles triangles, and scalene triangles. These concepts do not have meaning on their own but only have meaning in relation to the concept of similarity. For example, students can understand that an equilateral triangle is a triangle with three equal angles even though the corresponding sides of the two triangles have different lengths because this concept can be related to the concept of similarity they already possess. Learning obstacles that arise in the learning flow due to didactic design in this similarity material will cause students to experience didactical obstacles.

Regarding online-based learning materials, interactive online-based learning materials can be used to explain the concept of similarity. By using online-based learning materials, students can directly visualize the concept of similarity in the real world, making it easier for them to understand the concept. Students can use online learning applications to visualize equilateral triangles in front of them, then measure the angles to prove that the concept of similarity is correct. Students can also use online learning applications to construct equilateral triangles with materials around them, helping them understand the concept in a more enjoyable and interactive way.

Interactive online learning materials offer diverse content formats that provide integrated access. Educators can respond to various learning obstacles and student needs. Students, on the other hand, enjoy flexibility in terms of time and space when accessing knowledge, contributing to their cognitive development. From a philosophical standpoint, this phenomenon exemplifies the utilization of extended cognition, as it benefits the enhancement of human cognition (Pritchard, 2018).

By designing appropriate online-based learning materials, students can have a better learning experience and overcome learning barriers they may face. The use of technology in learning can help improve students' motivation, enhance their understanding of concepts, and increase their engagement in the learning process. Therefore, it is important to continuously develop and implement innovative and effective online-based learning materials to enhance students' online learning experiences.

4. CONCLUSION

The Hypothetical Learning Trajectory (HLT) is a crucial approach in instructional design to achieve higher understanding. In this study, an HLT was developed for the concept of similarity based on several stages in the Prospective DDR phase of Didactical Design Research. The resulting HLT consists of four learning stages designed to ensure comprehensive understanding of the concept of similarity and its application in real-world contexts. The implementation of the HLT in teaching the concept of similarity is expected to help students overcome common difficulties, such as a lack of initial understanding of scale and magnification concepts, difficulties in comprehending the relationship between ratios and scale factors previously learned, and a lack of relevant task sequencing related to the application of similarity in everyday life. In this HLT, the importance of student-centered approaches, the use of active learning strategies, and interactive student engagement are key factors. Moreover, the use of online learning materials plays a crucial role in enhancing the students' learning experience. Interactive online learning materials provide integrated access through various content formats, offering students flexibility in accessing knowledge and enabling faster responses to learning obstacles and students' needs. The implications of this research are that a carefully designed HLT approach, coupled with effective online learning materials, can enhance students' understanding of the concept of similarity and their ability to apply it in real-world situations. In the long term, this can have a positive impact on students' achievement in mathematics and their ability to think critically, connect concepts to everyday life, and solve problems effectively.

5. REFERENCES

- Aaron, L. S., & Lipton, T. (2018). Digital distraction: Shedding light on the 21st-century college classroom. *Journal of Educational Technology Systems*, 46(3), 363–378.
- Agra, G., Formiga, N. S., Oliveira, P. S. de, Costa, M. M. L., Fernandes, M. das G. M., & Nóbrega, M. M. L. da. (2019). Analysis of the concept of Meaningful Learning in light of the Ausubel's Theory. *Revista Brasileira de Enfermagem*, 72, 248–255.
- Aljawarneh, S. A. (2020). Reviewing and exploring innovative ubiquitous learning tools in higher education. *Journal of Computing in Higher Education*, 32, 57–73. <https://doi.org/10.1007/s12528-019-09207-0>
- Amaral-Schio, R., & Hollebrands, K. (2021). *An Analysis of The Concept of Similarity Presented in Textbooks in Brazil and The United States*.

- Andrews-Larson, C., Wawro, M., & Zandieh, M. (2017). A hypothetical learning trajectory for conceptualizing matrices as linear transformations. *International Journal of Mathematical Education in Science and Technology*, 48(6), 809–829.
- Audi, R. (2011). *Epistemology: A Contemporary Introduction to the Theory of Knowledge*. Routledge.
- Auh, Y., & Sim, H. R. (2019). Uses of social network topology and network-integrated multimedia for designing a large-scale open learning system: case studies of unsupervised featured learning platform Design in South Korea. *Multimedia Tools and Applications*, 78, 5445–5462. <https://doi.org/10.1007/s11042-018-6658-1>
- Bakker, A., & Smit, J. (2018). Using hypothetical learning trajectories in design research. In *Design Research in Education* (pp. 255–271). Routledge.
- Barnett, J., McPherson, V., & Sandieson, R. M. (2013). Connected teaching and learning: The uses and implications of connectivism in an online class. *Australasian Journal of Educational Technology*, 29(5).
- Bjorklund, D. F. (2022). *Children's thinking: Cognitive development and individual differences*. Sage publications.
- Bosch, M., Gascón, J., & Trigueros, M. (2017). Dialogue between theories interpreted as research praxeologies: the case of APOS and the ATD. *Educational Studies in Mathematics*, 95(1), 39–52. <https://doi.org/10.1007/s10649-016-9734-3>
- Carstens, K. J., Mallon, J. M., Bataineh, M., & Al-Bataineh, A. (2021). Effects of Technology on Student Learning. *Turkish Online Journal of Educational Technology-TOJET*, 20(1), 105–113.
- Castro, R. (2019). Blended learning in higher education: Trends and capabilities. *Education and Information Technologies*, 24(4), 2523–2546.
- Chevallard, Y., & Bosch, M. (2020). Anthropological theory of the didactic (ATD). *Encyclopedia of Mathematics Education*, 53–61.
- Clivaz, S., & Miyakawa, T. (2020). The effects of culture on mathematics lessons: an international comparative study of a collaboratively designed lesson. *Educational Studies in Mathematics*, 105(1), 53–70. <https://doi.org/10.1007/s10649-020-09980-1>
- Cox, D. C. (2013). Similarity in middle school mathematics: At the crossroads of geometry and number. *Mathematical Thinking and Learning*, 15(1), 3–23.
- Dakhi, O., Jama, J., & Irfan, D. (2020). Blended learning: a 21st century learning model at college. *International Journal Of Multi Science*, 1(08), 50–65.
- Decano, R. S. (2017). Cognitive development of college students and their achievement in geometry: an evaluation using Piaget's theory and Van Hiele's levels of thinking. *American Journal of Applied Sciences*, 14(190), 899–911.
- Dengel, A., & Mägdefrau, J. (2018). Immersive learning explored: Subjective and objective factors influencing learning outcomes in immersive educational virtual environments. *2018 IEEE International Conference on Teaching, Assessment, and Learning for Engineering (TALE)*, 608–615.
- Duschl, R., Maeng, S., & Sezen, A. (2011). Learning progressions and teaching sequences: A review and analysis. *Studies in Science Education*, 47(2), 123–182.
- Fägerstam, E. (2014). High school teachers' experience of the educational potential of outdoor teaching and learning. *Journal of Adventure Education & Outdoor Learning*, 14(1), 56–81.
- Ferri, F., Grifoni, P., & Guzzo, T. (2020). Online learning and emergency remote teaching: Opportunities and challenges in emergency situations. *Societies*, 10(4), 86.

- Fitriyani, F., Sunaryati, T., & Surya, V. M. K. (2023). Implementation Of Project-Based Learning Oriented To The Merdeka Learning Curriculum In The Form Of A Pancasila Student Profile With Global Diversity. *Buana Pendidikan: Jurnal Fakultas Keguruan Dan Ilmu Pendidikan Unipa Surabaya*, 19(1), 115–124. <https://doi.org/10.36456/bp.vol19.no1.a6832>
- Ghanizadeh, A., Al-Hoorie, A. H., & Jahedizadeh, S. (2020). *Higher order thinking skills in the language classroom: A concise guide*. Springer.
- Gillett-Swan, J. (2017). The challenges of online learning: Supporting and engaging the isolated learner. *Journal of Learning Design*, 10(1), 20–30. <https://doi.org/10.5204/jld.v9i3.293>
- Hadi, A., Marniati, M., Ngindana, R., Kurdi, M. S., Kurdi, M. S., & Fauziah, F. (2023). New Paradigm of Merdeka Belajar Curriculum in Schools. *AL-ISHLAH: Jurnal Pendidikan*, 15(2). <https://doi.org/10.35445/alishlah.v15i2.3126>
- Haleem, A., Javaid, M., Qadri, M. A., & Suman, R. (2022). Understanding the role of digital technologies in education: A review. *Sustainable Operations and Computers*.
- Hutchison, E. D. (2018). *Dimensions of human behavior: Person and environment*. Sage Publications.
- Jupri, A., Usdiyana, D., & Sispiyati, R. (2019). Designing an algebra learning sequence: the case of operations on algebraic expressions. *Journal of Physics: Conference Series*, 1280(4), 42038.
- Kazi, S., & Galanaki, E. (2019). Piagetian theory of cognitive development. *The Encyclopedia of Child and Adolescent Development*, 1–11.
- Kebritchi, M., Lipschuetz, A., & Santiago, L. (2017). Issues and challenges for teaching successful online courses in higher education: A literature review. *Journal of Educational Technology Systems*, 46(1), 4–29.
- Khan, A., Egbue, O., Palkie, B., & Madden, J. (2017). Active learning: Engaging students to maximize learning in an online course. *Electronic Journal of E-Learning*, 15(2), pp107-115.
- Kilag, O. K. T., Ignacio, R., Lumando, E. B., Alvez, G. U., Abendan, C. F. K., Quiñanola, N. M. P., & Sasan, J. M. (2022). *ICT Integration in Primary Classrooms in the Light of Jean Piaget's Cognitive Development Theory*.
- Ko, S., & Rossen, S. (2017). *Teaching online: A practical guide*. Taylor & Francis.
- Krishnan, I. A., Ching, H. S., Ramalingam, S., Maruthai, E., Kandasamy, P., De Mello, G., Munian, S., & Ling, W. W. (2020). Challenges of learning English in 21st century: Online vs. traditional during Covid-19. *Malaysian Journal of Social Sciences and Humanities (MJSSH)*, 5(9), 1–15.
- Kuncoro, K. S., Sukiyanto, S., Irfan, M., Amalia, A. F., Pusporini, W., Wijayanti, A., & Widodo, S. A. (2022). Peningkatan Literasi Digital Guru Guna Mengatasi Permasalahan Pembelajaran di Era Pandemi Covid-19. *Abdi Wiralodra: Jurnal Pengabdian Kepada Masyarakat*, 4(1), 17–34. <https://doi.org/10.31943/abdi.v4i1.50>
- Lestari, K. I., Arcana, I. N., Susetyo, A. E., & Kuncoro, K. S. (2022). Development of Online Learning Quiz and Educational Game Using Word Walls in Mathematics for Grade 10. *INSANIA: Jurnal Pemikiran Alternatif Kependidikan*, 27(2), 145–159. <https://doi.org/10.24090/insania.v27i2.6924>
- Long, T., Cummins, J., & Waugh, M. (2017). Use of the flipped classroom instructional model in higher education: instructors' perspectives. *Journal of Computing in Higher Education*, 29, 179–200.

- Martin, F., & Bolliger, D. U. (2018). Engagement matters: Student perceptions on the importance of engagement strategies in the online learning environment. *Online Learning, 22*(1), 205–222.
- Mcleod, S. (2022). Vygotsky's sociocultural theory of cognitive development. Retrieved from *Simply Psychology: <https://www.simplypsychology.org/Vygotsky.html>*.
- Myers, S. (2021). Jerome Bruner: An Overview of Key Ideas. *Jerome Bruner, Meaning Making and Education for Conflict Resolution*.
- O'Halloran, K. L., Beezer, R. A., & Farmer, D. W. (2018). A new generation of mathematics textbook research and development. *ZDM, 50*(5), 863–879.
- OECD. (2019). *PISA 2018 Results (Volume I) : What Students Know and Can Do*. PISA, OECD Publishing. <https://doi.org/10.1787/5f07c754-en>
- Parisa, M., Arcana, I. N., Susetyo, A. E., & Kuncoro, K. S. (2023). Pengembangan Kuis dan Game Edukasi Menggunakan Wordwall pada Pembelajaran Daring Pertidaksamaan Nilai Mutlak Bentuk Linier. *Jurnal THEOREMS (The Original Research of Mathematics), 7*(2), 167–180. <https://doi.org/10.31949/th.v7i2.4351>
- Prabowo, A., & Juandi, D. (2020). Analisis situasi didaktis dalam pembelajaran matematika berbantuan ICT pada siswa SMP. *Pythagoras: Jurnal Pendidikan Matematika, 15*(1), 1–12.
- Pritchard, D. (2018). *What is this thing called knowledge?* Routledge.
- Puspita, R., Yani, E., Dinnisa, K., Kusumaningrum, B., Kuncoro, K. S., Ayuningtyas, A. D., & Irfan, M. (2022). Interactive Math Path: Permainan Ular Tangga Berbasis Etnomatematika. *UNION: Jurnal Ilmiah Pendidikan Matematika, 10*(1), 93–102. <https://doi.org/10.30738/union.v10i1.12139>
- Qian, Y., & Lehman, J. (2017). Students' misconceptions and other difficulties in introductory programming: A literature review. *ACM Transactions on Computing Education (TOCE), 18*(1), 1–24.
- Rahiem, M. D. H. (2020). The emergency remote learning experience of university students in Indonesia amidst the COVID-19 crisis. *International Journal of Learning, Teaching and Educational Research, 19*(6), 1–26.
- Rapanta, C., Botturi, L., Goodyear, P., Guàrdia, L., & Koole, M. (2020). Online university teaching during and after the Covid-19 crisis: Refocusing teacher presence and learning activity. *Postdigital Science and Education, 2*, 923–945.
- Reid, M., & Reid, S. (2017). Learning to Be a Math Teacher: What Knowledge Is Essential?. *International Electronic Journal of Elementary Education, 9*(4), 851–872.
- Rusydiah, E. F., Purwati, E., & Prabowo, A. (2020). How to use digital literacy as a learning resource for teacher candidates in Indonesia. *Cakrawala Pendidikan, 39*(2), 305–318.
- Seago, N., Jacobs, J., Driscoll, M., Matassa, M., & Callahan, M. (2013). Developing teachers' knowledge of a transformations-based approach to geometric similarity. *Mathematics Teacher Educator, 2*(1), 74–85. <https://doi.org/10.5951/mathteaceduc.2.1.0074>
- Serrano, D. R., Dea-Ayuela, M. A., Gonzalez-Burgos, E., Serrano-Gil, A., & Lalatsa, A. (2019). Technology-enhanced learning in higher education: How to enhance student engagement through blended learning. *European Journal of Education, 54*(2), 273–286. <https://doi.org/10.1111/ejed.12330>
- Shinno, Y., & Mizoguchi, T. (2021). Theoretical approaches to teachers' lesson designs involving the adaptation of mathematics textbooks: two cases from kyouzai kenkyuu in Japan. *ZDM - Mathematics Education, 53*(6), 1387–1402. <https://doi.org/10.1007/s11858-021-01269-8>

- Simsek, A., Bretscher, N., Clark-Wilson, A., & Hoyles, C. (2022). Teachers' classroom use of dynamic mathematical technology to address misconceptions about geometric similarity. *Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)*.
- Simsek, A., & Clark-Wilson, A. (2018). Teacher knowledge for teaching geometric similarity with technology: A review of literature. *Edited by: Hans-Georg Weigand, Alison Clark-Wilson, Ana Donevska*, 249.
- Sun, J. C., Wu, Y., & Lee, W. (2017). The effect of the flipped classroom approach to OpenCourseWare instruction on students' self-regulation. *British Journal of Educational Technology*, 48(3), 713–729.
- Suryadi, D. (2019a). *Landasan Filosofis Penelitian Desain Didaktis (DDR)*. Pusat Pengembangan DDR Indonesia.
- Suryadi, D. (2019b). *Penelitian Desain Didaktis (DDR) dan Implementasinya*. Gapura Press.
- Susanto, D., Sihombing, S., Radjawane, M. M., Wardani, A. K., Kurniawan, T., Candra, Y., & Mulyani, S. (2022). *Matematika*. Pusat Perbukuan Badan Standar, Kurikulum, dan Asesmen Pendidikan Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi.
- Szymkowiak, A., Melović, B., Dabić, M., Jeganathan, K., & Kundi, G. S. (2021). Information technology and Gen Z: The role of teachers, the internet, and technology in the education of young people. *Technology in Society*, 65, 101565. <https://doi.org/10.1016/j.techsoc.2021.101565>
- Takeuchi, H., & Shinno, Y. (2020). Comparing the Lower Secondary Textbooks of Japan and England: a Praxeological Analysis of Symmetry and Transformations in Geometry. *International Journal of Science and Mathematics Education*, 18(4), 791–810. <https://doi.org/10.1007/s10763-019-09982-3>
- Tang, T., Abuhmaid, A. M., Olaimat, M., Oudat, D. M., Aldhaeebi, M., & Bamanger, E. (2020). Efficiency of flipped classroom with online-based teaching under COVID-19. *Interactive Learning Environments*, 1–12.
- Taranto, E., Robutti, O., & Arzarello, F. (2020). Learning within MOOCs for mathematics teacher education. *ZDM - Mathematics Education*, 52(7), 1439–1453. <https://doi.org/10.1007/s11858-020-01178-2>
- Utami, N. S., Prabawanto, S., & Priatna, N. (2022). A didactical design for introducing the concepts in algebraic forms using the theory of praxeology. *Beta: Jurnal Tadris Matematika*, 15(1), 53–71.
- Wang, J., Shimada, A., Oi, M., Ogata, H., & Tabata, Y. (2020). Development and evaluation of a visualization system to support meaningful e-book learning. *Interactive Learning Environments*, 1–18.
- Wijayanti, D., & Aufa, D. N. (2020). Picturing Textbook on Exponent Equations Based on Praxeology Organization. *2nd Social and Humaniora Research Symposium (SoRes 2019)*, 494–498.
- Wu, J.-Y., & Cheng, T. (2019). Who is better adapted in learning online within the personal learning environment? Relating gender differences in cognitive attention networks to digital distraction. *Computers & Education*, 128, 312–329.
- Wulandari, D., Arcana, I. N., & Kuncoro, K. S. (2022). Pengembangan Instagram reels pembelajaran pokok bahasan persamaan garis lurus untuk SMP. *UNION: Jurnal Ilmiah Pendidikan Matematika*, 10(1), 1–14. <https://doi.org/10.30738/union.v10i1.12138>
- Zainuddin, Z., Shujahat, M., Haruna, H., & Chu, S. K. W. (2020). The role of gamified e-quizzes on student learning and engagement: An interactive gamification solution for a formative assessment system. *Computers & Education*, 145, 103729.