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From Scholarly Knowledge to Knowledge to be Taught: The Case of Vector Introduction

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ABSTRACTS

The incorporation of the contextualization and tangible instances in the diffusion of vector concepts in a textbook can lead to students forming a concept image that differs from the scientific understanding due to their perceptions and actions towards the learning environment. This study aims to expose potential learning obstacles in vector introduction caused by external didactic transposition through the study of textbook analysis. Meta-discourse served as the methodological analysis. Two textbooks were analyzed based on concept declaration, the changing environment, and didactic phenomena. Based on the concept declaration, the presence of an epistemological obstacle is a potential learning barrier. Additionally, changes in the learning environment can contribute to ontogenic obstacles. In conclusion, the analysis of the three textbooks reveals that potential learning obstacles encompass both ontogenic and epistemological aspects.

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1. INTRODUCTION

Didactic transposition refers to the process of transforming scholarly knowledge into knowledge to be taught, which is then further transformed into taught knowledge and learned knowledge. The concept of didactic transposition was first introduced by French mathematician and educator Yves Chevallard in the 1980's (Chevallard & Bosch, 2020). Each knowledge in different institutions undergoes a shift in meaning when transposed (Chevallard, 2007). However, the ultimate goal of this transposition process is to bridge the gap of knowledge between different institutions. The first transposition issues arise when scholars' a priori knowledge is externally transposed into knowledge to be taught (Suryadi, 2019). This is because mathematical knowledge produced by mathematicians is considered a priori knowledge philosophically (Maudy, 2023). Nevertheless, the consideration of the diffusion and acquisition process when transposing this a priori knowledge into knowledge to be taught results in mathematical knowledge becoming a posteriori knowledge.

The topic of vectors, which is covered in the linear algebra course of computer science education programs, holds significant practical relevance. It finds applications in various domains, including data representation such as text (Shah, 2021), images (Wang et al., 2021), sound (Gosztolya, 2020), preferences (Hui et al., 2022) and more (Chaudhuri et al., 2021; Panda and Ray, 2022). As a result, there is a wide range of textbooks available that serve as references for teaching materials, each with their own approaches to presenting concepts and applying them in different contexts. However, it is important to note that the incorporation of contextualization and concrete examples in teaching vector concepts, as highlighted by Suryadi (2019), can lead to students forming a concept image that may deviate from the scientific understanding due to their perceptions and interactions within the learning environment.

Previous research has exposed several problems. Bækkelie (2019) has shown that there was a significant difference of concept image of rank between the two groups of students concerning their understanding of the concept rank due to the presentation of concepts that only focus on the procedure of searching for a rank without giving meaning to the rank itself. In addition, the research conducted by Lyse-Olsen and Fleischmann (2022) revealed that logical relations in the context of learning about eigen theory have an impact on the formalism and preciseness of their descriptions. Furthermore, the variations in the task's context concerning linear independence given to in-service teachers lead to misunderstandings and errors that arise due to misinterpretations of the solutions to the system of equations and challenges faced in creating the general vector equation and performing scalar multiplication (Mutambara & Bansilal, 2022).

To address this issue, several researchers have proposed a solution to this problem by recommending the association of students' concept image or comprehension of the concept with the challenges, such as difficulties (Bianchini et al., 2019; Mutambara & Bansilal, 2022) or pedagogic (Bækkelie, 2019; Bouhjar et al., 2021; Açikyildiz & Temel, 2021) as partial problem, encountered during their learning process. However, addressing students' learning challenges is not enough, as the root cause of the didactic system problems lies in the learning obstacles (Brousseau, 2002). Therefore, it is necessary to identify the learning obstacles that lead to these difficulties.

Therefore, the objective of this study is to analyze various textbooks on applied linear algebra regarding the introduction of vectors. From a didactic perspective, the focus is on examining whether the conceptual structure and thought processes developed in these textbooks pose any potential learning barriers. Additionally, from an epistemological standpoint, the aim is to determine whether the material design, including the context and presentation methods, aligns with the principles of knowledge construction. This evaluation will shed light on potential learning obstacles that may arise from the external didactic transposition in vector introduction, as observed through textbook analysis. Overall, this research aims to contribute to the identification and understanding of possible challenges

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encountered in the teaching and learning of vector concepts through an examination of textbooks.

2. METHODS

In this section, we will discuss the methodology of our research project, including the procedures we used for data collection, the instruments we employed to gather data, and the methods we used to analyze the data to answer our research questions. The primary data analyzed were taken from undergraduate basic or elementary linear algebra and structure algebra textbooks as a resource to construct body knowledge of vector space as scholarly knowledge. Each book was chosen as a representative of linear algebra and structure algebra textbooks published since 1998 in a five-year period. Meanwhile, the book Wilson and Gibbs (1901) was chosen due to historical reasons for the emergence of the vector concept itself.

As a representative of knowledge to be taught, three books were selected as data sources, namely: Anton et al. (2019) and Boyd and Vandenberghe (2018). Anton et al. (2019) was chosen due to widely used as a reference book in linear algebra courses, especially in engineering programs in Indonesia. Meanwhile, the diversity of contexts in the field of computer science in Boyd and Vandenberghe (2018) is the reason for selecting this book.

The focus in this research is the concept of vectors as an object in Mathematics, as well as vectors as an element of a vector space. Some initial properties that are sometimes inseparable, such as the components, length, direction of vectors, and vector operation, are also included in the discussion. Meanwhile, other concepts related to the properties of a vector space such as subspaces, linear combinations, linear independence, basis, and dimension are not included in this research.

Analytical data was collected to refer to what Kang and Kilpatrick (1992) used in didactic transposition in mathematics textbooks, which are concept declaration, the changing environment, and didactic phenomenon. The analysis of concept declaration will examine how a textbook emphasizes a concept as social acknowledgement and legitimation of the knowledge to be taught. The contextualization and personalization of knowledge will be analyzed through the explanation of the changing environment in scholarly knowledge to the textbook. Furthermore, didactic phenomena, the case of metacognitive shift, will be analyzed as potential learning obstacles that may arise from the presentation of these books.

3. RESULTS AND DISCUSSION

a. Epistemological insight on the introduction to vector concepts

In this study, we investigated the role of vectors as a foundational concept in scholarly knowledge across various fields, including mathematics, physics, engineering, and computer science. This session focuses on the evolution of vector theory as a scholarly knowledge, tracing its historical origins and its structural concept as a mathematical object. The result can be found in Table 1.

Table 1. Structural concept of vector as scholarly knowledge

No	Definitions	References
1a	A vector is a quantity which is considered as possessing direction as well as magnitude. A scalar is a quantity which is considered as possessing direction but no direction.	Wilson and Gibbs (1901)

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Table 1. Continued

No	Definitions	References
1b	Vector is a list of objects. If the object consists of n elements belonging to a field F, then an n-dimensional column vector is an $n \times 1$ matrix over F.	Matthews (1998)
2a	The collection of all n–dimensional column vectors, denoted by F^n, is vector space.	Matthews (1998)
2b	Suppose R is a ring and M is an additive abelian group. The statement that M is a *right* $R - module$ means there is a scalar multiplication $M \times R \to M$ satisfying $(a_1 + a_2)r = a_1r + a_2r$ $(m, r) \to mr$ $\quad a(r_1 + r_2) = ar_1 + ar_2$ $a(r_1 \cdot r_2) = (ar_1)r_2$ $a1 = a$ for all a, a_1, $a_2 \in M$ and r, r_1, $r_2 \in R$. In the case where R is a field F, then $F - modules$ are called vector spaces.	Connell (2002)
2c	Let V be a set with an associative, commutative, binary operation, $+$, called addition. Let $(F, \oplus, \odot)$ be a field with a mapping $\cdot : F \times V \to V$. We say that $(V, F, +, \cdot)$ is a vector space, or that V is a vector space over F under + and ·, provided the following axioms hold. (a) There is **0** in V such that $\boldsymbol{v} + 0 = \boldsymbol{v}$ for all $\boldsymbol{v}$ in V. (b) For each $\boldsymbol{v}$ in V there is $-\boldsymbol{v}$ in V such that $\boldsymbol{v} + (-\boldsymbol{v}) = 0$. (c) $c \cdot (\boldsymbol{v} + \boldsymbol{w}) = c \cdot \boldsymbol{v} + c \cdot \boldsymbol{w}$, for all c in F and all $\boldsymbol{v}, \boldsymbol{w}$ in V. (d) $(a \oplus b) \cdot \boldsymbol{v} = a \cdot \boldsymbol{v} + b \cdot \boldsymbol{v}$, for all a, b in F and $\boldsymbol{v}$ in V. (e) $(a \odot b) \cdot \boldsymbol{v} = a \cdot (b \cdot \boldsymbol{v})$, for all a, b in F and $\boldsymbol{v}$ in V. (f) $1 \cdot \boldsymbol{v} = \boldsymbol{v}$, for all $\boldsymbol{v}$ in V.	Dillon (2022)

Wilson and Gibbs (1901) presented the concept of vectors by drawing attention to the different types of quantities that can be represented using a single number (e.g., mass, time) versus those that require both magnitude and direction (e.g., force, velocity). They formally define vector in Definition 1a. This idea is also discussed in other works such as Kwak and Hong (2004) and Friedberg et al. (2019), although they may not provide a formal definition. Additionally, another works introduces vectors as an alternative term for referring to row matrices or column matrices, using the terms "row vector" or "column vector" (Connell, 2002; Oliveira, 2022), particularly within the context of systems of linear equations.

Tran (2022) defines a vector as a collection of numbers, aligning with Matthews (1998), who describes a vector as a list of objects, whether they are unknowns, variables, or numbers. Specifically, when the object comprises n elements from a field F, an n-dimensional column vector is represented as an $n \times 1$ matrix over F, as stated in Definition 1b. The set of all n-dimensional column vectors is denoted by F^n. Andrilli and Hecker (2010) provide a more precise formal definition of a vector, where a real n-vector is defined as an ordered n-tuple of real numbers.

In other references (Dillon, 2022; Groves et al., 2015; Liesen & Mehrmann, 2015; Lang, 2010), the definition of a vector is not presented as an independent object but rather as an

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element of a vector space. Most books discussing vectors include formal or informal definitions of vector spaces. However, Wilson and Gibbs (1901) did not explicitly use the term "vector space."

The concept of a vector space is generally constructed based on the structures of groups, rings, fields, and modules. Various versions of the vector space definition can be found in different sources, such as Definitions 2a-2c. Definition 2a considers a vector space as a collection of vectors, where the discussion revolves around a single set F, and F^n is seen as a generalized form of F. As a result, the operations involved in vector spaces are those applicable to F.

The definition of a vector space has evolved to distinguish between the set of vectors and the scalars involved, leading to the introduction of two set structures, V and F. According to Connell (2002) in Definition 2b, a vector space is defined as a module over a field. It is important to note that the plus sign is used ambiguously, serving as both addition in M and addition in R. When the field R is equivalent to a field F, the F-modules are referred to as vector spaces (Connell, 2002). While Golan (2007), Lang (2010), Liesen and Mehrmann (2015), and Friedberg et al. (2019) do not explicitly mention the concept of a module in their books, their definitions of vector spaces exhibit structural similarities. In Dillon (2022) Definition 2c, there is a more distinct difference between the addition operation applicable to V and F. Unlike Kwak and Hong (2004), Hefferon (2006), Andrilli and Hecker (2010), Groves et al. (2015), Tran (2022), and Oliveira (2022), who have similar sentence structures, they do not explicitly state that F must be a field. Generally, the vector spaces discussed in these books are within the context of real or complex scalars.

b. Textbook References Analysis using Meta-Discourse

As mentioned in the methodology section, three reference books were analyzed as representatives of the knowledge to be taught. The summaries of the declared concepts of vector as well as vector spaces can be found in Table 2 and Table 3.

Table 2. Concept declaration in Anton et al. (2019)

<table>
<tr><th>Concept</th><th>Structural concept organization</th></tr>
<tr><td rowspan="2">Euclidean Vector Spaces</td><td>Geometric vector in 2-Space and 3-Space
A vector is an arrow in 2-space or 3-space. The direction of the arrow head specifies the direction of the vector and the length of the arrow specifies the magnitude. The tail of the arrow is called the initial point of the vector and the tip the terminal point (Figure 1a). A vector $\boldsymbol{v}$ has initial point A and terminal point B then $\boldsymbol{v} = \overrightarrow{AB}$ (Figure 1b).

Figure 1. Geometric vector in 2-Space and 3-Space illustration</td></tr>
<tr><td>n-Space
If n is a positive integer, then an ordered n-tuple is a sequence of n real numbers $(\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_n)$. The set of all ordered n-tuples is called real n-space and is denoted by R^n.</td></tr>
</table>

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Table 2. Continued

Concept	Structural concept organization
General Vector Space	Let V be an arbitrary nonempty set of objects for which two operations are defined: addition and multiplication by numbers called scalars. By addition we mean a rule for associating with each pair of objects $\boldsymbol{u}$ and $\boldsymbol{v}$ in V an object $\boldsymbol{u}+\boldsymbol{v}$, called the sum of $\boldsymbol{u}$ and $\boldsymbol{v}$; by scalar multiplication we mean a rule for associating with each scalar k and each object $\boldsymbol{u}$ in V an object $k\boldsymbol{u}$, called the scalar multiple of $\boldsymbol{u}$ by k. If the following axioms are satisfied by all objects $\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}$ in V and all scalars k and m, then we call V a vector space and we call the objects in V vectors. 1. If $\boldsymbol{u}$ and $\boldsymbol{v}$ are objects in V, then $\boldsymbol{u}$ +v is in V. 2. $\boldsymbol{u}+\boldsymbol{v}=\boldsymbol{v}+\boldsymbol{u}$. 3. $\boldsymbol{u}+(\boldsymbol{v}+\boldsymbol{w}) = (\boldsymbol{u}+\boldsymbol{v})+\boldsymbol{w}$. 4. There exists an object in V, called the zero vector, that is denoted by **0** and has the property that $\mathbf{0}+\boldsymbol{u}=\boldsymbol{u}+\mathbf{0}=\boldsymbol{u}$ for all $\boldsymbol{u}$ in V. 5. For each $\boldsymbol{u}$ in V, there is an object $-\boldsymbol{u}$ in V, called a negative of $\boldsymbol{u}$, such that $\boldsymbol{u}+(-\boldsymbol{u})=\boldsymbol{u}-\boldsymbol{u}=\mathbf{0}$. 6. If k is any scalar and $\boldsymbol{u}$ is any object in V, then $k\boldsymbol{u}$ is in V. 7. $k(\boldsymbol{u}+\boldsymbol{v}) = k\boldsymbol{u}+k\boldsymbol{v}$. 8. $(k+m)\boldsymbol{u} = k\boldsymbol{u}+m\boldsymbol{u}$ 9. $k(m\boldsymbol{u}) = (km)\boldsymbol{u}$ 10. $1\boldsymbol{u} = \boldsymbol{u}$ Vector spaces with real scalars will be called real vector spaces and those with complex scalars will be called complex vector spaces.

Following the perspectives expressed in Dillon (2022), Groves et al. (2015), Liesen and Mehrmann (2015) and Lang (2010), the concept declaration of the vector definition presented in Anton et al. (2019) tends to emphasize the notion of a vector as an element of a vector space. Particularly, within this context, the initial introduction of vectors places emphasis on 2-spaces and 3-spaces from a geometric standpoint, which is then extended to encompass n-space and vector spaces in general.

In relation to the conceptual declaration presented in Anton et al. (2019), there are two noteworthy observations regarding the use of notation and the structure of the scalar set. The notation employed for vectors and scalars remains consistent throughout; however, the utilization of addition notation can be perplexing. This potential confusion may pose learning challenges for computer science students as they navigate distinguishing between the notation for vector addition and scalar addition within the definition of a general vector space. In terms of the required structure of the scalar set, Anton et al. (2019) do not explicitly stipulate that the scalar set must be a field similar to Kwak and Hong (2004), Hefferon (2006), Andrilli and Hecker (2010), Groves et al. (2015), Tran (2022), and Oliveira (2022). Nonetheless, towards the end of the definition, it is clarified that the scalar set utilized corresponds to either the set of real numbers or the set of complex numbers. These two aspects—the ambiguity surrounding summation notation and the necessary structure of the scalar set—can potentially give rise to epistemological obstacles.

Referring to changing environment within the given context in Anton et al. (2019), there are various contexts to consider, such as experimental data, storage and warehousing, electrical circuits, graphical images, economics, mechanical systems, and the RGB color

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model, for example, to illustrate the implementation and application of n-space. Furthermore, to provide a more detailed explanation of vector spaces in general, examples are given in the context of n-real vector space, infinite sequences of real numbers, $m \times n$ matrices, and real-valued functions. The tasks and exercises are presented in the context of geometric vectors in 2-Space and 3-Space, sequences of n real numbers, 2×2 matrices, infinite sequences of real numbers, $m \times n$ matrices, and real-valued functions. Out of the 58 tasks provided, only 7 of them are not based on the contexts of 2-space, 3-space, or n-space. This observation highlights the occurrence of a metacognitive shift, where the didactic device emphasizes a geometric perspective in 2-space and 3-space, while adopting an algebraic perspective in n-space.

Table 3. Concept declaration in Boyd and Vanderberghe (2018)

Concept	Structural concept organization
Vector	1. A vector is an ordered finite list of numbers. The elements (or entries, coefficients, components) of a vector are the values in the array. The size (also called dimension or length) of the vector is the number of elements it contains. A vector of size n is called an n-vector. The ith element of the vector a is denoted a_i. The numbers or values of the elements in a vector are called scalars. 2. The set of all real numbers is written as R, and the set of all real n-vectors is denoted R^n, so $a \in R^n$ is another way to say that a is an n-vector with real entries.

Contrary to Anton et al. (2019), Boyd and Vanderberghe (2018) present a different concept declaration where vectors are defined as an ordered finite list of numbers. However, this differs from the declarations in Tran (2022) and Matthews (1998), which state that vectors are an ordered list of objects comprising n elements from a field F. Boyd and Vanderberghe (2018) specifically limit the components of the objects to be real numbers. Additionally, the term "vector space" is not mentioned as a formal definition for the set of given vectors. This discrepancy raises an epistemological obstacle when considering the structural framework of a vector space as defined by mathematicians. Nevertheless, it is worth noting that Boyd and Vanderberghe (2018) book is primarily aimed at engineering students who require vectors as a means of representing data that can be understood by computers.

In addition to the concept declaration variations, the changing environment within the given context introduces a wide range of contexts to consider. These contexts encompass diverse areas such as location and displacement, color, quantities, portfolio management, values across a population, proportions, time series analysis, financial transactions, images, videos, word count analysis, customer purchases, subsets or occurrences, features or attributes, bill of materials, market dynamics, as well as lines and segments (Boyd & Vanderberghe, 2018). Each context provides a unique perspective on the application and understanding of vectors in different domains. By exploring these diverse contexts, students can gain a comprehensive understanding of how vectors are utilized to analyze and represent various real-world phenomena, reinforcing their knowledge and problem-solving skills in computer science.

Furthermore, the tasks and exercises presented within the concept declaration cover a broad range of contexts, expanding on the understanding of vectors in specific applications. These contexts include the n-real vector space, energy usage analysis, daily cash flow

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management, annual dollar value purchases, portfolio optimization, bill of materials for project management, monochrome image processing, rainfall analysis in specific locations, Boolean set operations, profit and sales analysis, symptom assessment in medical patients, course score evaluation, word count analysis, cash value calculations, age distribution in a population, and quantities of materials (Boyd & Vanderberghe, 2018). By providing tasks in these varied contexts, students can develop a deeper understanding of how vectors are applied to solve problems in diverse fields, reinforcing the connection between vector concepts and their real-world applications, particularly in computer science-related scenarios.

Based on the analysis of two textbook references as representatives and several undergraduate mathematics textbooks as representatives of scholarly knowledge, it is found that there are three different definitions related to the formal introduction of vectors. Firstly, a vector is defined as a quantity that has magnitude and direction. This definition emphasizes the physical interpretation of vectors, representing quantities such as force, velocity, or displacement. Secondly, a vector is defined as an ordered list of objects, specifically numbers. This definition focuses on the algebraic representation of vectors, where each element in the list corresponds to a component of the vector. Lastly, a vector is defined as a member of a vector space, highlighting the structural perspective of vector spaces. In this context, vectors are seen as elements of a mathematical structure with specific properties and operations.

Based on the concept declarations presented in the analyzed textbooks, there is a potential for both ontogenic and epistemological obstacles. The different definitions of vectors can lead to ontogenic obstacles, as students may encounter challenges in understanding and reconciling these diverse perspectives. The shift in conceptualization from physical quantities to algebraic representations and mathematical structures can pose difficulties in connecting and applying these concepts in different contexts.

Furthermore, the variations in definitions and the absence of a unified approach to introducing vectors can create epistemological obstacles. Students may struggle to develop a coherent understanding of the fundamental properties and operations associated with vectors, particularly when transitioning between different textbooks or encountering different contexts. The lack of consistency in terminology and conceptual frameworks can hinder students' ability to generalize their knowledge and apply it effectively in problem-solving situations.

4. CONCLUSION

In conclusion, the analysis of the concept declarations in the examined textbooks reveals the existence of multiple definitions of vectors, highlighting the potential for ontogenic and epistemological obstacles in learning. The presence of multiple definitions of vectors has significant implications for mathematics education. It necessitates the need for educators to carefully select and present a coherent and comprehensive definition that aligns with the learning goals and the broader mathematical framework. A clear and consistent definition of vectors will not only help students develop a solid foundation but also enable them to apply vector concepts effectively in different mathematical and real-world contexts.

To address the potential ontogenic and epistemological obstacles, it is recommended that educators adopt a balanced approach that integrates different perspectives on vectors. This can involve providing a broad overview of the various definitions and their corresponding contexts, highlighting the connections and relationships between them. By explicitly addressing the different definitions and their implications, educators can help students develop a more nuanced understanding of vectors and enhance their metacognitive awareness. Additionally, it is crucial to scaffold students' learning experiences by gradually

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introducing more complex contexts and applications of vectors. This can include engaging students in problem-solving tasks that require them to analyze and interpret vector quantities in diverse situations, such as computer graphics or data analysis. By providing opportunities for students to explore and manipulate vectors in different contexts, educators can help them develop a more robust understanding of the underlying concepts and overcome potential obstacles.

5. AUTHORS' NOTE

The authors declare that there is no conflict of interest regarding the publication of this article. Authors confirmed that the paper was free of plagiarism.

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