



# Models for Interactions in Boundary Layers at Rotational Motions in Noncircular Orbits: The Concept for Teaching Science

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## ABSTRACTS

The motions of a boomerang, a tennis ball with spin, and a flock of birds are studied with the assumption that, occasionally, capture into a noncircular orbit (NCO) is ruling the behavior. This approximation will be used in applications when the orbit is almost circular, but the angular velocity varies due to the interaction between orbital and sidereal motion. A format with decreasing radius is exploited, to model drop, due to spin. Collection and exchange by generalized conjugated forces from energy are extracted with a semi-derivative, which appears to give no loss, but only contributions. It is proposed that the collection of energy in surrounding air layers of a certain shape may be beneficial for propulsion. Since the analyses were based on daily life conditions, this can be applied for a support media in teaching science.

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## 1. INTRODUCTION

In the first approximation for the classical mechanics at the free motion of a rigid body, the path for the center of mass and the rotation are decoupled, i.e. they are independently determined by initial conditions and the force system acting on the body. For the planetary system (integer ratios between sidereal and orbital motion), the moon rotates synchronously, which is a phenomenon not predicted by classical theory.

Here, we will consider a refinement and assume that the body together with air layers move in noncircular orbits (occasionally), both sidereal and orbital. The latter is the parabola for the center of mass. The modelling is applied to a tennis ball with spin and the boomerang.

Then, the results for energy and area measured are elaborated to model possible exchange and collection of energy at motion in air layers. For that purpose, the semi-derivative of an area measure is analysed. Such paths, extracted from energy variables, are found to materialize in flocks of flying birds. Since the analyses were based on daily life conditions, this can be applied for a support media in teaching science.

## 2. METHOD

The method was a derivation of some equations based on phenomena in daily life conditions. We focused on the condition in the boomerang, tennis ball, and eccentricity of flying bird condition.

## 3. RESULTS AND DISCUSSION

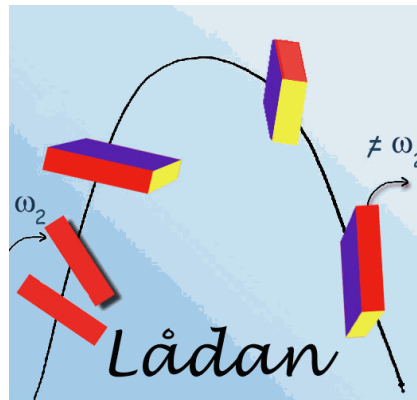
### 3.1. The Boomerang

The boomerang is a non-symmetric item (see **Figure 1**) (Smolarski & Saylor, 1988). It has 3 different inertia moments. The solution is given by the so-called Poinso't's construction (as known as Classical Mechanics) (see **Figure 2**). In practice/Netto, at rotation, possibly the in-plane mass is smeared and then the classical solution is a precession of the main rotation axis. Since it turns, it is probably a wobbling, as for a bicycle, i.e. a precession in two directions, where the amplitude is larger inwards, and this may be due to the actual non-symmetric shape and initial conditions. Possibly at the largest distance, the rotational velocity and orbital velocity get smaller such that the more 'chaotic' motion around 2-axis is governing. Then it may wobble and turn more distinct which makes the entire orbit more elongated than a circle, and connect presence to the previous parts by discrete dynamics, so-called Tti-memory (Strömberg, 2015).



**Figure 1.** Boomerangs, in various designs.

According to Poinsot's construction, the solutions are given by polehoods of the inertia ellipsoidal, which is in the frame of the rotational body (often known as the Lagrangian system). In a fixed system (Eulerian frame), the tangents at rotation are descriptive and these are analyzed within theories of geodetics.



**Figure 2.** Illustration of free motion of a rigid body, with 3 different inertia moments.

Next, an intuitive interpretation with functional expressions will be outlined. The tangents in a fixed Cartesian plane are given by contacts of the rolling inertia ellipsoid. Assume a format  $w = w_0 \exp(2(r_e/r_0) \sin(fw_0t))$ , for small ratios  $r_e/r_0$  at a linearization, we can write Equation [1]:

$$dw = d(w_0^2(r_e/r_0) \sin(fw_0t)) \quad (1)$$

Compared with real rolling contact with  $t$  in an interval  $[t_0, t_1]$ , it describes that the 'pressure' increases from a small value to a maximum and then decreases after which contact ceases. Such an assumption for each in-plane component gives that the precession may be estimated to be harmonic and the coordinates in the time interval determine the amplitudes.

### 3.2. Tennis Ball with Spin

For a circular motion, the differential line element ( $ds$ ) can be expressed in polar coordinates  $(r_0, v)$ , written in Equation [2]:

$$ds^2 = dr_0^2 + r_0^2 dv^2 \quad (2)$$

With Equation [1], a modification of the differential angle  $dv$  will be assumed. To describe the case when the eccentricity ( $r_e$ ) in the NCO is small, but other parameters (e.g.  $r_0$  and  $w_0$ ) vary, such that results  $w$  for the NCO is significant. Then, we can derive Equation [3]:

$$ds^2 = dr_0^2 (1 + (w_0 \Delta t 2(r_e/r_0) \sin(fw_0t))^2) + r_0^2 dv^2 \quad (3)$$

where  $\Delta t$  is a finite time interval, in which a finite angle is summed by the integration of Equation [1].

When approaching the peak in the orbital path, the sidereal rotation may experience a change in the  $r_0$  for its path (i.e. when the orbital path becomes more curved, with a decreasing radius if described with circles instead of a parabola).

The spin for a ball may change, such that a contribution to the angle and incremental radius is positive and negative. With the description of Equation [3], the resulting length increases for all cases. But there is no information about the direction. When downwards, it is known as a stop (or drop). This can be found in the tennis ball (see **Figure 3**). This occurs when the

sidereal rotation is opposite. In general, some such phenomena are known as the Magnus effect.

Possibly, a planetary coupling as integer ratios (e.g. -1:1 as Venus) is activated. If to satisfy that, the sidereal rotation decreases rapidly and the energy redistributes and rearranges into oscillating deformations of the ball and layers, this may cause it to not bounce as predicted while hitting the ground. However, that may mostly depend on the conditions of the ball and is a phenomenon for satellites, within the discipline of 'music for the planetary system' dating back to Pythagoras, as notified in.



**Figure 3.** Illustration of tennis.

### 3.3. Integration and Differentiation of Area Measures in the Eccentricity Zone

An 'in-between' smaller derive is of interest, e.g. some energy measures are quadratic. Next, the function for area measure will be analysed, to obtain a multiplicative split. Proposition: Consider the fractional derive (Tarasov, 2004). With  $\alpha=1/2$  and the argument being square  $x*x$ , the fractional derivative reads  $2/\pi^{1/2} \text{abs}(x)$ . This is the so-called semi-derivative (Eigel & Sturm, 2018).

*Remark.* We may note that normalization with a half circle, connects to an area. This motivates exploiting the semi-derivative to extract

- a (specific) variable in a quadratic energy measure
- a lower dimension from a measure in a layer

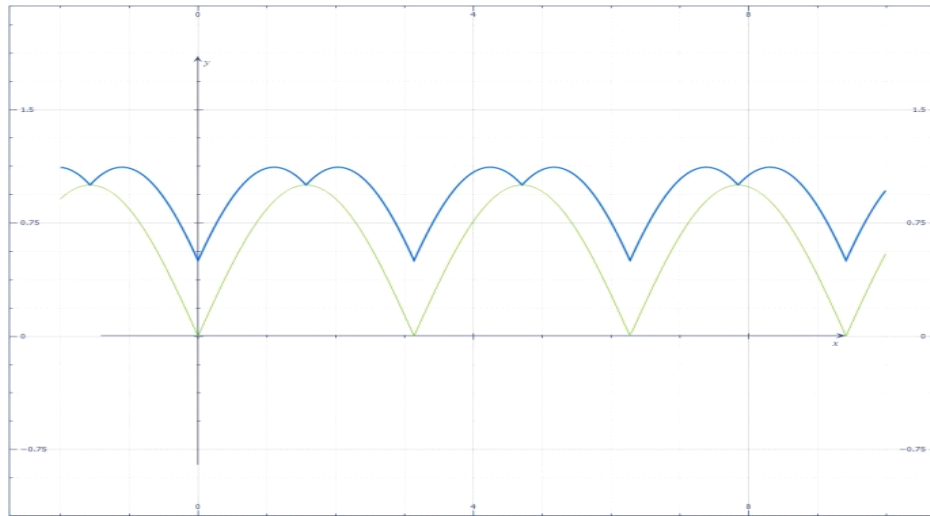
As an example, for the collection of energy, we consider a large number of birds flying high up in the sky (Figure 4), such that while observed from the ground, it looks almost like a one-dimensional elongated continuum of black smoke.



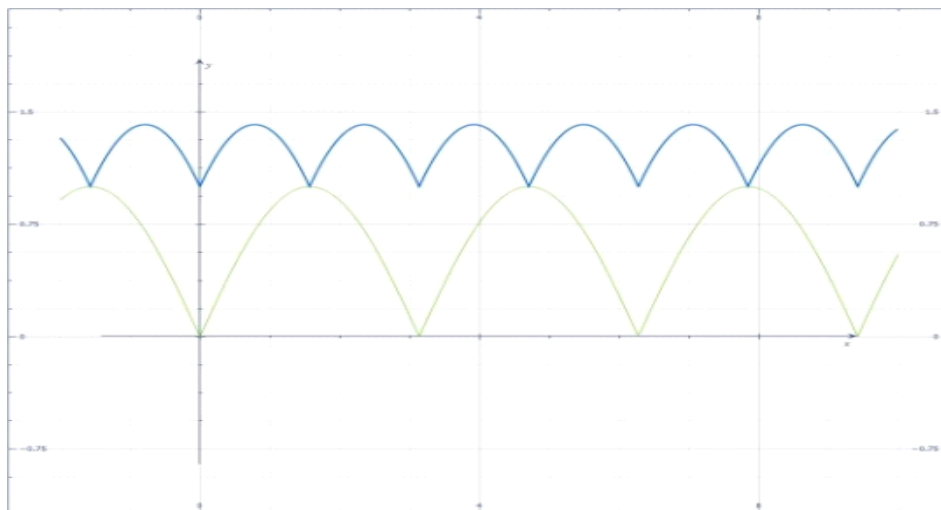
**Figure 4.** The flock of birds.

At first, they flew in two long rows. The first condition was straighter and the other curved. It is 3-5 times in the center and bows (see **Figures 5** and **6**). This is similar to the upper-most (see a blue curve in **Figure 6**). The Equation can be written as Equation [4]

$$f(x) = \text{abs}(\sin(x)) + 0.5 * \text{abs}(\cos(x)) \quad (4)$$



**Figure 5.** Lower curve green:  $\text{abs}(\sin(x))$ . Upper curve blue:  $\text{abs}(\sin(x)) + 0.5 * \text{abs}(\cos(x))$



**Figure 6.** Lower curve green:  $\text{abs}(\sin(x))$ . Upper curve blue:  $\text{abs}(\sin(x)) + \text{abs}(\cos(x))$

The modelling refers to the similarity of the row to the right on the photo and the blue curve in **Figure 5**. Eventually, they turned, and the formation rearranged into a large vertical circle. It is with possibly 1-5 birds in the thickness direction).

*Hypothesis:* 'Collection of energy (in an exchange with the atmosphere)'.

Above, and in previous texts, energy measures are assumed quadratic, e.g. as functionals of  $(r-r_0)^2$  from the area of an eccentricity zone, or proportional to  $w^2$ . The preliminaries above provide a background for that path like in **Figures 5** and **6** may be beneficial for collecting energy. It is a materialization of the Equation (4), if space is assumed flat locally, i.e. there are

local coordinates  $x$  proportional to  $\omega_0 t$  on flat Cartesian charts, and the continuation/extension forms a spatial axis with coordinates that are in Equation [4].

#### 4. CONCLUSION

To model interaction with air layers, refined models for the intrinsic dynamic of rotating bodies were proposed. In addition to the above results, a possible application for the celestial curves in **Figure 6**, will be discussed. In the example, we found that the exchange of energy while positive could be modeled with a semi-derivative. An application may be that with such a shape, the interaction with air layers could be enhanced, which is beneficial in aircraft working by electrical ionization. A forward net thrust may be modeled with a drop-in pressure at the front compared with behind. That may be accomplished with an ionization. Then, to maintain a motion, a normal state, or a state with increased pressure is necessary at the back of the vehicle, which requires a certain time scale of relaxation of the medium and/or interaction with the electric frame, compared with the time scale given by the velocity and the length of the vehicle. With the format of ideal gas law in a continuum with pressure, density, and temperature as variables, a pressure drop is present at the change of molecule mass.

#### 5. ACKNOWLEDGMENTS

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